Half-Angle Identities for Tangent
Practice Problems

1. Find the exact value of $\tan(15°)$.

2. Find the exact value of $\tan(-195°)$.

3. Find the exact value of $\tan\left(\frac{5\pi}{8}\right)$.

4. Find the exact value of $\tan\left(\frac{19\pi}{12}\right)$.

5. Find the exact value of $\tan(-202.5°)$.

6. Find the exact value of $\tan\left(\frac{x}{2}\right)$ if $\cot x = \frac{4}{3}, 180° < x < 270°$.

7. Find the exact value of $\tan\left(\frac{x}{2}\right)$ if $\sec x = \frac{8}{3}, 270° < x < 360°$.

Solutions

Note: Half-Angles for tangent can be found using any of the three formulas shown below. I suggest using the same one for every problem.

$$\tan\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos x}{1+\cos x}},\quad \tan\left(\frac{x}{2}\right)=\frac{\sin x}{1+\cos x},\quad \tan\left(\frac{x}{2}\right)=\frac{1-\cos x}{\sin x}$$

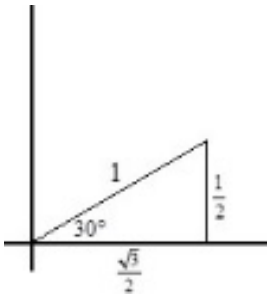
1. Find the exact value of $\tan(15°)$.

$$\tan(15°)=\tan\left(\frac{30°}{2}\right)=\frac{\sin(30°)}{1+\cos(30°)}$$

$$=\frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}=\frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}=\frac{1}{2}\cdot\frac{2}{2+\sqrt{3}}$$

$$=\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{1}=2-\sqrt{3}$$



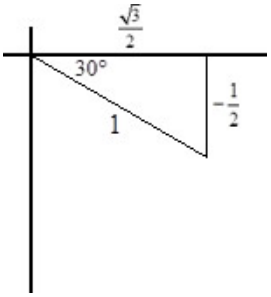
2. Find the exact value of $\tan(-195°)$.

$$\tan(-195°)=\tan(165°)=\tan\left(\frac{330°}{2}\right)=\frac{\sin(330°)}{1+\cos(330°)}$$

$$=\frac{-\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}=\frac{-\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}=-\frac{1}{2}\cdot\frac{2}{2+\sqrt{3}}$$

$$=\frac{-1}{2+\sqrt{3}}=\frac{-1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{-2+\sqrt{3}}{1}=-2+\sqrt{3}$$



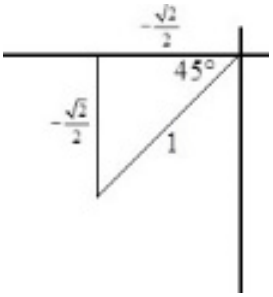
3. Find the exact value of $\tan\left(\frac{5\pi}{8}\right)$.

$$\tan\left(\frac{5\pi}{8}\right)=\tan(112.5°)=\tan\left(\frac{225°}{2}\right)=\frac{\sin(225°)}{1+\cos(225°)}$$

$$=\frac{-\frac{\sqrt{2}}{2}}{1+\left(-\frac{\sqrt{2}}{2}\right)}=\frac{-\frac{\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}=-\frac{\sqrt{2}}{2}\cdot\frac{2}{2-\sqrt{2}}$$

$$=\frac{-\sqrt{2}}{2-\sqrt{2}}=\frac{-\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}$$

$$=\frac{-2\sqrt{2}-2}{2}=-\sqrt{2}-1$$



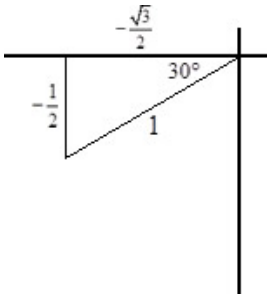
4. Find the exact value of $\tan\left(\frac{19\pi}{12}\right)$.

$$\tan\left(\frac{19\pi}{12}\right)=\tan(285°)=\tan\left(\frac{570°}{2}\right)=\frac{\sin(570°)}{1+\cos(570°)}$$

$$=\frac{-\frac{1}{2}}{1+\left(-\frac{\sqrt{3}}{2}\right)}=\frac{-\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}=-\frac{1}{2}\cdot\frac{2}{2-\sqrt{3}}$$

$$=\frac{-1}{2-\sqrt{3}}=\frac{-1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$=\frac{-2-\sqrt{3}}{1}=-2-\sqrt{3}$$



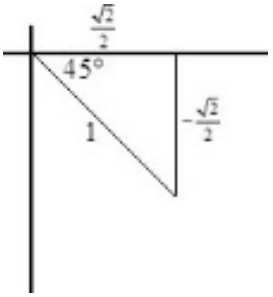
5. Find the exact value of $\tan(-202.5°)$.

$$\tan(-202.5°)=\tan(157.5°)=\tan\left(\frac{315°}{2}\right)=\frac{\sin(315°)}{1+\cos(315°)}$$

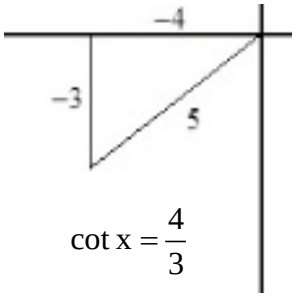
$$=\frac{-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}=\frac{-\frac{\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}=-\frac{\sqrt{2}}{2}\cdot\frac{2}{2+\sqrt{2}}$$

$$=\frac{-\sqrt{2}}{2+\sqrt{2}}=\frac{-\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$=\frac{-2\sqrt{2}+2}{2}=-\sqrt{2}+1$$



6. Find the exact value of $\tan\left(\frac{x}{2}\right)$ if $\cot x = \frac{4}{3}, 180° < x < 270°$.



$\cot x = \frac{4}{3}$

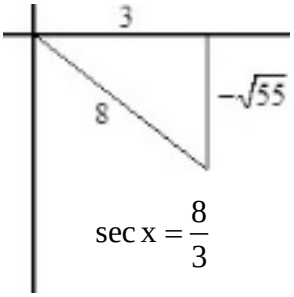
$\Rightarrow \sin x = \frac{-3}{5}$

$\Rightarrow \cos x = \frac{-4}{5}$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x} = \frac{\frac{-3}{5}}{1+\left(\frac{-4}{5}\right)}$$

$$= \frac{\frac{-3}{5}}{\frac{1}{5}} = \frac{-3}{5}\cdot\frac{5}{1} = -3$$

7. Find the exact value of $\tan\left(\frac{x}{2}\right)$ if $\sec x = \frac{8}{3}, 270° < x < 360°$.



$\sec x = \frac{8}{3}$

$\Rightarrow \sin x = \frac{-\sqrt{55}}{8}$

$\Rightarrow \cos x = \frac{3}{8}$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x} = \frac{\frac{-\sqrt{55}}{8}}{1+\frac{3}{8}}$$

$$= \frac{\frac{-\sqrt{55}}{8}}{\frac{11}{8}} = \frac{-\sqrt{55}}{8}\cdot\frac{8}{11} = \frac{-\sqrt{55}}{11}$$