Solving Quadratics by the Square Root Principle

The square root principle is a technique that can be used to solve quadratics, but in order to solve a quadratic using the square root principle the problem must be in the correct form. To solve a quadratic using the square root principle the quadratic must be in vertex form, \(a(x - h)^2 + k\). In other words, the quadratic must have a perfect square such as \((x + 5)^2\) and no other x’s in the problem.

Consider the following two examples:

\[
2(x - 4)^2 + 7 = 19, \quad \text{this problem can be solved using the square root principle because there is a perfect square, } (x - 4)^2, \text{ and there is only one x in the problem.}
\]

\[
2(x - 4)^2 + 7x = 19, \quad \text{this problem cannot be solved using the square root principle because even though there is a perfect square, } (x - 4)^2, \text{ there is a second x in the problem. This problem would need to be solved using a different technique such as the quadratic formula.}
\]

If a problem is in the correct form, the most efficient way to solve the problem is by using the square root principle. So, what is the square root principle?

<table>
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<th>Square Root Principle</th>
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<td>If (x^2 = k), then (x = \pm \sqrt{k})</td>
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Basically, the square root principle says that if \(x^2\) equals some number, \(k\), then to find the solutions all we need to do is take the square root of \(k\). The reason for the ± is that an equation involving \(x^2\) must have two solutions, so the two solutions are \(\pm \sqrt{k}\). Most students forget to include the ± when using the square root principle, so try not to forget the ±.

Here are the steps we can follow to solve quadratics using the square root principle:

- **Step 1:** Isolate the perfect square.
- **Step 2:** Take the square root of each side. Don’t forget the ±.
- **Step 3:** Simplify the radical.
- **Step 4:** Get x by itself.

Now let’s look at some examples of solving quadratics using the square root principle.

**Example 1:** Solve \((x - 5)^2 = 36\)

**Step 1:** Isolate the perfect square.

In this case the perfect square is already by itself.

**Step 2:** Take the square root of each side. Don’t forget the ±.

\[
(x - 5)^2 = 36 \rightarrow \sqrt{(x - 5)^2} = \pm \sqrt{36} \rightarrow x - 5 = \pm 6
\]

**Step 3:** Simplify the radical.

\[
x - 5 = \pm 6 \rightarrow x = 5 \pm 6
\]

**Step 4:** Get x by itself.

\[
x - 5 = \pm 6 \rightarrow x = 5 \pm 6
\]

The final answer is \(x = 11\) or \(x = -1\). We simplified the problem here because \(5 + 6 = 11\) and \(5 - 6 = -1\) and this is easily done without sing decimals or needing a calculator to find the answer.
Example 2: Solve $3(x + 7)^2 - 2 = 22$

**Step 1**: Isolate the perfect square.

We need to isolate the perfect square by adding 2 and dividing by 3.

$3(x + 7)^2 - 2 = 22 \rightarrow (x + 7)^2 = 8$

**Step 2**: Take the square root of each side. Don’t forget the $\pm$.

$(x + 7)^2 = 8 \rightarrow \sqrt{(x + 7)^2} = \pm\sqrt{8} \rightarrow x + 7 = \pm\sqrt{8}$

**Step 3**: Simplify the radical.

$x + 7 = \pm\sqrt{8} \rightarrow x + 7 = \pm2\sqrt{2}$

**Step 4**: Get $x$ by itself.

$x + 7 = \pm2\sqrt{2} \rightarrow x = -7 \pm 2\sqrt{2}$

The final answer is $x = -7 \pm 2\sqrt{2}$.

Example 3: Solve $2(x - 4)^2 + 7 = -20$

**Step 1**: Isolate the perfect square.

We need to isolate the perfect square by subtracting 7 and dividing by 2.

$2(x - 4)^2 + 7 = -20 \rightarrow (x - 4)^2 = \frac{-27}{2}$

**Step 2**: Take the square root of each side. Don’t forget the $\pm$.

$(x - 4)^2 = \frac{-27}{2} \rightarrow \sqrt{(x - 4)^2} = \pm\sqrt{\frac{-27}{2}} \rightarrow x - 4 = \pm\sqrt{\frac{-27}{2}}$

**Step 3**: Simplify the radical.

The problem is written this way to remind us to rationalize the denominator.

$x - 4 = \pm\frac{\sqrt{-27}}{\sqrt{2}} \rightarrow x - 4 = \pm\frac{\sqrt{-27}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow x - 4 = \pm\frac{\sqrt{-54}}{2} \rightarrow x - 4 = \pm\frac{3\sqrt{6}}{2}$

**Step 4**: Get $x$ by itself.

$x - 4 = \pm\frac{3\sqrt{6}}{2} \rightarrow x = 4 \pm \frac{3\sqrt{6}}{2}$

The final answer is $x = 4 \pm \frac{3\sqrt{6}}{2}$ or $x = 4 \pm \frac{3\sqrt{6}}{2}$. i.
Example 4: Solve \(3(x + 9)^2 - 11 = 41\)

**Step 1:** Isolate the perfect square.

We need to isolate the perfect square by adding 11 and dividing by 3.

\[
3(x + 9)^2 - 11 = 41 \implies (x + 9)^2 = \frac{52}{3}
\]

**Step 2:** Take the square root of each side. Don’t forget the ±.

\[
(x + 9)^2 = \frac{52}{3} \implies \sqrt{(x + 9)^2} = \pm \frac{\sqrt{52}}{\sqrt{3}} \implies x + 9 = \pm \frac{\sqrt{52}}{\sqrt{3}}
\]

**Step 3:** Simplify the radical.

The problem is written this way to remind us to rationalize the denominator.

\[
x + 9 = \pm \frac{\sqrt{52}}{\sqrt{3}} \implies x + 9 = \pm \frac{\sqrt{52}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \implies x + 9 = \pm \frac{\sqrt{156}}{3} \implies x + 9 = \pm \frac{2\sqrt{39}}{3}
\]

**Step 4:** Get x by itself.

\[
x + 9 = \pm \frac{2\sqrt{39}}{3} \implies x = -9 \pm \frac{2\sqrt{39}}{3}
\]

The final answer is \(x = -9 \pm \frac{2\sqrt{39}}{3}\).

Example 5: Solve \(4(x - 6)^2 - 35 = 14\)

**Step 1:** Isolate the perfect square.

We need to isolate the perfect square by adding 35 and dividing by 4.

\[
4(x - 6)^2 - 35 = 14 \implies (x - 6)^2 = \frac{49}{4}
\]

**Step 2:** Take the square root of each side. Don’t forget the ±.

\[
(x - 6)^2 = \frac{49}{4} \implies \sqrt{(x - 6)^2} = \pm \frac{\sqrt{49}}{\sqrt{4}} \implies x - 6 = \pm \frac{7}{4}
\]

**Step 3:** Simplify the radical.

\[
x - 6 = \pm \frac{7}{4} \implies x - 6 = \pm \frac{7}{4}
\]

**Step 4:** Get x by itself.

\[
x - 6 = \pm \frac{7}{4} \implies x = 6 \pm \frac{7}{4}
\]

The final answer is \(x = \frac{31}{4}\) or \(x = \frac{17}{4}\).