

Solving Quadratics by the Quadratic Formula

The quadratic formula is a technique that can be used to solve quadratics, but in order to solve a quadratic using the quadratic formula the problem must be in the correct form. To solve a quadratic using the quadratic formula the quadratic must be in the form $ax^2 + bx + c = 0$. In other words, the quadratic must be in descending order (highest power to lowest power) and equal to zero making it easy to identify the values of a, b, and c to plug into the quadratic formula.

The quadratic formula is generally used to solve all quadratics in the form $ax^2 + bx + c = 0$ and will find the solution to the problem every time. However, there are times when other solving techniques are faster and should be considered. Consider the following problems:

$x^2 - 7x - 18 = 0 \rightarrow$ This problem can be solved using the quadratic formula, but it would be faster to solve the problem by factoring. Generally, factoring should be considered if the leading coefficient is a 1, meaning the problem is in the form $x^2 + bx + c = 0$.

$2(x - 5)^2 - 7 = 0 \rightarrow$ This problem could be solved using the quadratic formula, but we would need to simplify the problem first in order to get the problem in the correct form. A better option would be to use the square root principle.

Here is the quadratic formula:

$$\text{Quadratic Formula}$$
$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is a formula that should be memorized; the quadratic formula is used frequently throughout mathematics and should never be forgotten! When solving a problem using the quadratic formula here are the steps we should follow for each problem:

- Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.
- Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.
- Step 3: Simplify the numbers within the quadratic formula.
- Step 4: Simplify the radical and reduce/simplify to get the final answer.

Next we will look at several example of how to use the quadratic formula to solve quadratics.

Example 1: Solve $12x^2 + 7x = 12$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$12x^2 + 7x = 12 \rightarrow 12x^2 + 7x - 12 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = 12, b = 7, \text{ and } c = -12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4(12)(-12)}}{2(12)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-7 \pm \sqrt{7^2 - 4(12)(-12)}}{2(12)} \rightarrow x = \frac{-7 \pm \sqrt{625}}{24}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{-7 \pm \sqrt{625}}{24} \rightarrow x = \frac{-7 \pm 25}{24} \rightarrow x = \frac{3}{4} \text{ or } x = -\frac{4}{3}$$

Note: In this example the radical disappeared and the final answers were simple fractions which means that this problem could have been solved by factoring, but since we already have the answer at this point it makes little difference.

Example 2: Solve $-4x^2 - 9x = -12$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$-4x^2 - 9x = -12 \rightarrow -4x^2 - 9x + 12 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = -4, b = -9, \text{ and } c = 12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-4)(12)}}{2(-4)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-4)(12)}}{2(-4)} \rightarrow x = \frac{9 \pm \sqrt{273}}{-8}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{9 \pm \sqrt{273}}{-8} \rightarrow \text{This problem cannot be simplified.}$$

Note: In Step 2, if we use a calculator to find $(-9)^2 = 81$ it must be in parenthesis because if we type -9^2 we will get -81 and the answer would end up being incorrect.

Example 3: Solve $x^2 + 25 = 8x$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$x^2 + 25 = 8x \rightarrow x^2 - 8x + 25 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = 1, b = -8, \text{ and } c = 25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(25)}}{2(1)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(25)}}{2(1)} \rightarrow x = \frac{8 \pm \sqrt{-36}}{2}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{8 \pm \sqrt{-36}}{2} \rightarrow x = \frac{8 \pm 6i}{2} \rightarrow x = 4 \pm 3i$$

Example 4: Solve $6x^2 = -6x + 9$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$6x^2 = -6x + 9 \rightarrow 6x^2 + 6x - 9 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = 6, b = 6, \text{ and } c = -9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-6 \pm \sqrt{6^2 - 4(6)(-9)}}{2(6)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(6)(-9)}}{2(6)} \rightarrow x = \frac{-6 \pm \sqrt{252}}{12}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{-6 \pm \sqrt{252}}{12} \rightarrow x = \frac{-6 \pm 6\sqrt{7}}{12} \rightarrow x = \frac{-1 \pm \sqrt{7}}{2}$$

Note: In this example the final answer could be reduced or simplified because 6 divided evenly into all three parts of the fraction. Consider the two examples shown below.

$$x = \frac{6 \pm 4\sqrt{13}}{10} \rightarrow \text{This can be reduced/simplified.} \rightarrow x = \frac{3 \pm 2\sqrt{13}}{5}$$

$$x = \frac{8 \pm 5\sqrt{11}}{6} \rightarrow \text{This cannot be reduced/simplified.}$$

\rightarrow There isn't a number that divided evenly into 8, 5, and 6.

Example 5: Solve $-3x^2 + 8x = 9$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$-3x^2 + 8x = 9 \rightarrow -3x^2 + 8x - 9 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = -3, b = 8, \text{ and } c = -9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-8 \pm \sqrt{8^2 - 4(-3)(-9)}}{2(-3)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(-3)(-9)}}{2(-3)} \rightarrow x = \frac{-8 \pm \sqrt{-44}}{-6}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{-8 \pm \sqrt{-44}}{-6} \rightarrow x = \frac{-8 \pm 2i\sqrt{11}}{-6} \rightarrow x = \frac{4}{3} \pm \frac{\sqrt{11}}{3}i$$

Note: In this example the final answer is a complex number which means the final answer needs to be written in the form $a + bi$. The real part of the solution needs to be separate from the imaginary part of the solution.

Example 6: Solve $(4x - 3)^2 = 5x$

Step 1: Simplify the problem to get the problem in the form $ax^2 + bx + c = 0$.

$$(4x - 3)^2 = 5x \rightarrow 16x^2 - 24x + 9 = 5x \rightarrow 16x^2 - 29x + 9 = 0$$

Step 2: Identify the values of a, b, and c, then plug them into the quadratic formula.

$$a = 16, b = -29, \text{ and } c = 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(16)(9)}}{2(16)}$$

Step 3: Simplify the numbers within the quadratic formula.

$$x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(16)(9)}}{2(16)} \rightarrow x = \frac{30 \pm \sqrt{265}}{32}$$

Step 4: Simplify the radical and reduce/simplify to get the final answer.

$$x = \frac{30 \pm \sqrt{265}}{32} \rightarrow \text{This problem cannot be simplified.}$$

Note: In this example the problem could not be solved using the square root principle because there are two x 's in the problem.