Using Cramer’s Rule to Solve Two Equations with Two Unknowns
Practice Problems

1. Use Cramer’s Rule to solve:
   \[ \begin{align*}
   3x - 4y &= 6 \\
   5x - 3y &= -1
   \end{align*} \]

2. Use Cramer’s Rule to solve:
   \[ \begin{align*}
   6x - 3y &= 11 \\
   4x + 7y &= -6
   \end{align*} \]

3. Use Cramer’s Rule to solve:
   \[ \begin{align*}
   -2x + 5y &= 9 \\
   3x + 4y &= -2
   \end{align*} \]

4. Use Cramer’s Rule to solve:
   \[ \begin{align*}
   6x + 9y &= -7 \\
   5x + 8y &= -2
   \end{align*} \]

5. Use Cramer’s Rule to solve:
   \[ \begin{align*}
   4x - 7y &= 16 \\
   5x - 9y &= -7
   \end{align*} \]

Answers

1. \((-2, -3)\)

2. \(\left( \frac{59}{54}, -\frac{40}{27} \right)\)

3. \((-2, 1)\)

4. \(\left( -\frac{38}{3}, \frac{23}{3} \right)\)

5. \(\left( \frac{193}{22}, \frac{30}{11} \right)\)
Detailed Solutions

1. Use Cramer’s Rule to solve: \[\begin{aligned}3x - 4y &= 6 \\5x - 3y &= -1\end{aligned}\]

   **Step 1:** Find the determinant, \(D\), by using the \(x\) and \(y\) values from the problem.
   \[D = \begin{vmatrix} 3 & -4 \\ 5 & -3 \end{vmatrix} = -9 - (-20) = 11\]

   **Step 2:** Find the determinant, \(D_x\), by replacing the \(x\)-values in the first column with the values after the equal sign leaving the \(y\) column unchanged.
   \[D_x = \begin{vmatrix} 6 & -4 \\ -1 & -3 \end{vmatrix} = -18 - 4 = -22\]

   **Step 3:** Find the determinant, \(D_y\), by replacing the \(y\)-values in the second column with the values after the equal sign leaving the \(x\) column unchanged.
   \[D_y = \begin{vmatrix} 3 & 6 \\ 5 & -1 \end{vmatrix} = -3 - 30 = -33\]

   **Step 4:** Use Cramer’s Rule to find the values of \(x\) and \(y\).
   \[x = \frac{D_x}{D} = \frac{-22}{11} = -2\]
   \[y = \frac{D_y}{D} = \frac{-33}{11} = -3\]

   The answer written as an ordered pair is \((-2, -3)\).

2. Use Cramer’s Rule to solve: \[\begin{aligned}6x - 3y &= 11 \\4x + 7y &= -6\end{aligned}\]

   **Step 1:** Find the determinant, \(D\), by using the \(x\) and \(y\) values from the problem.
   \[D = \begin{vmatrix} 6 & -3 \\ 4 & 7 \end{vmatrix} = 42 - (-12) = 54\]
Step 2: Find the determinant, $D_x$, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

\[
D_x = \begin{vmatrix} 11 & -3 \\ -6 & 7 \end{vmatrix} = 77 - 18 = 59
\]

Step 3: Find the determinant, $D_y$, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

\[
D_y = \begin{vmatrix} 6 & -11 \\ 4 & -6 \end{vmatrix} = -36 - 44 = -80
\]

Step 4: Use Cramer’s Rule to find the values of x and y.

\[
x = \frac{D_x}{D} = \frac{59}{54}
\]

\[
y = \frac{D_y}{D} = \frac{-80}{54} = \frac{-40}{27}
\]

The answer written as an ordered pair is \((\frac{59}{54}, \frac{-40}{27})\).

3. Use Cramer’s Rule to solve: $\begin{cases} -2x + 5y = 9 \\ 3x + 4y = -2 \end{cases}$

Step 1: Find the determinant, $D$, by using the x and y values from the problem.

\[
D = \begin{vmatrix} -2 & 5 \\ 3 & 4 \end{vmatrix} = -8 - 15 = -23
\]

Step 2: Find the determinant, $D_x$, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

\[
D_x = \begin{vmatrix} 9 & 5 \\ -2 & 4 \end{vmatrix} = 36 - (-10) = 46
\]

Step 3: Find the determinant, $D_y$, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

\[
D_y = \begin{vmatrix} -2 & 9 \\ 3 & -2 \end{vmatrix} = 4 - 27 = -23
\]
Step 4: Use Cramer’s Rule to find the values of x and y.

\[
x = \frac{D_x}{D} = \frac{46}{-23} = -2
\]

\[
y = \frac{D_y}{D} = \frac{-23}{-23} = 1
\]

The answer written as an ordered pair is \((-2, 1)\).

4. Use Cramer’s Rule to solve:

\[
\begin{align*}
6x + 9y &= -7 \\
5x + 8y &= -2
\end{align*}
\]

Step 1: Find the determinant, D, by using the x and y values from the problem.

\[
D = \begin{vmatrix} 6 & 9 \\ 5 & 8 \end{vmatrix} = 48 - 45 = 3
\]

Step 2: Find the determinant, \(D_x\), by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

\[
D_x = \begin{vmatrix} -7 & 9 \\ -2 & 8 \end{vmatrix} = -56 - (-18) = -38
\]

Step 3: Find the determinant, \(D_y\), by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

\[
D_y = \begin{vmatrix} 6 & -7 \\ 5 & -2 \end{vmatrix} = -12 - (-35) = 23
\]

Step 4: Use Cramer’s Rule to find the values of x and y.

\[
x = \frac{D_x}{D} = \frac{-38}{3} = -\frac{38}{3}
\]

\[
y = \frac{D_y}{D} = \frac{23}{3}
\]

The answer written as an ordered pair is \(\left(-\frac{38}{3}, \frac{23}{3}\right)\).
5. Use Cramer’s Rule to solve: \[ \begin{align*}
4x - 7y &= 16 \\
2x - 9y &= -7
\end{align*} \]

**Step 1:** Find the determinant, \( D \), by using the \( x \) and \( y \) values from the problem.

\[
D = \begin{vmatrix}
4 & -7 \\
2 & -9
\end{vmatrix} = -36 - (-14) = -22
\]

**Step 2:** Find the determinant, \( D_x \), by replacing the \( x \)-values in the first column with the values after the equal sign leaving the \( y \) column unchanged.

\[
D_x = \begin{vmatrix}
16 & -7 \\
-7 & -9
\end{vmatrix} = -144 - 49 = -193
\]

**Step 3:** Find the determinant, \( D_y \), by replacing the \( y \)-values in the second column with the values after the equal sign leaving the \( x \) column unchanged.

\[
D_y = \begin{vmatrix}
4 & 16 \\
2 & -7
\end{vmatrix} = -28 - 32 = -60
\]

**Step 4:** Use Cramer’s Rule to find the values of \( x \) and \( y \).

\[
x = \frac{D_x}{D} = \frac{-193}{-22} = \frac{193}{22}
\]

\[
y = \frac{D_y}{D} = \frac{-60}{-22} = \frac{30}{11}
\]

The answer written as an ordered pair is \( \left( \frac{193}{22}, \frac{30}{11} \right) \).