

Solving Exponential Equations

Deciding How to Solve Exponential Equations

When asked to solve an exponential equation such as $2^{x+6} = 32$ or $5^{2x-3} = 18$, the first thing we need to do is to decide which way is the “best” way to solve the problem. Some exponential equations can be solved by rewriting each side of the equation using the same base. Other exponential equations can only be solved by using logarithms. How do we decide what is the “best” way to solve an exponential equation? The key is to look at the base of the exponential equation and determine if the each side of the problem can be rewritten using the same base.

If we consider the problem $2^{x+6} = 32$, the base of the exponent is 2 and we need to decide if we can rewrite the number 32 using only the number 2. In this case it is possible to write the number 32 using only 2's, $32 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$. This means that the “best” way to solve the problem $2^{x+6} = 32$ is to rewrite the problem using the base 2. If we consider the problem $5^{2x-3} = 18$, the base of the exponent is 5 and we need to decide if we can rewrite the number 18 using only the number 5. In this case it is impossible to write the number 18 using only 5's, $18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$. This means that the “best” and only way to solve the problem $5^{2x-3} = 18$ is to use logarithms.

Solving Exponential Equations with the Same Base

After deciding the best way to solve an exponential equation is by rewriting each side of the equation using the same base, what is next?

Solving Exponential Equations with the Same Base

$$\text{If } B^M = B^N, \text{ then } M = N.$$

This statement simply says that if the bases are the same then the exponents must be the same. To solve an exponential equation with the same base, first we need to rewrite the problem using the same base and after getting the bases the same we can drop the bases and set the exponents equal to each other.

Let's finish solving the problem $2^{x+6} = 32$ from before. In this problem we have already seen that both 2 and 32 can be written using the number 2. If we continue from there we get:

$$2^{x+6} = 2^5$$

Rewrite the problem using the same base.

$$x + 6 = 5$$

Since the bases are the same, we can drop the bases and set the exponents equal to each other.

$$x = -1$$

Finish solving by subtracting 6 from each side.

Therefore, the solution to the problem $2^{x+6} = 32$ is $x = -1$.

There is another example, solve $9^{2x-5} = 27$.

$$9^{2x-5} = 27$$

Determine if 9 and 27 can be written using the same base. In this case both 9 and 27 can be written using the base 3.

$$(3^2)^{2x-5} = 3^3$$

Rewrite the problem using the same base.

$$3^{4x-5} = 3^3$$

Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.

$$4x - 5 = 3$$

Since the bases are the same, we can drop the bases and set the exponents equal to each other.

$$x = 2$$

Finish solving by adding 5 to each side and then dividing each side by 4.

Therefore, the solution to the problem $9^{2x-5} = 27$ is $x = 2$.

Now that we have looked at a couple of examples of solving exponential equations with the same base, let's list the steps for solving exponential equations that have the same base.

Steps for Solving Exponential Equations with the Same Base

Step 1: Determine if the numbers can be written using the same base. If so, go to Step 2. If not, stop and use Steps for Solving an Exponential Equation with Different Bases.

Step 2: Rewrite the problem using the same base.

Step 3: Use the properties of exponents to simplify the problem.

Step 4: Once the bases are the same, drop the bases and set the exponents equal to each other.

Step 5: Finish solving the problem by isolating the variable.

Solving Exponential Equations with the Different Bases

After deciding the only way to solve an exponential equation is to use logarithms, what is next? The next step is to take the common logarithm or natural logarithm of each side. By taking the logarithm of each side, we can use the properties of logarithms, specifically property 5 from our list of properties, to rewrite the exponential problem as a multiplication problem. After changing the problem from an exponential problem to a multiplication problem using the properties of logarithms we will be able to finish solving the problem.

Let's finish solving the problem $5^{2x-3} = 18$ from before. In this problem we have already seen that it is impossible to rewrite the numbers 5 and 18 using the same base, so we must use logarithms. Continuing on here is what we get:

$$\log(5^{2x-3}) = \log(18)$$

Take the common logarithm or natural logarithm of each side.

$$(2x - 3)(\log 5) = \log 18$$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$$2x - 3 = \frac{\log 18}{\log 5}$$

Divide each side by $\log 5$.

$$2x - 3 \approx 1.795889$$

Use a calculator to find $\log 18$ divided by $\log 5$. Round the answer as appropriate, these answers will use 6 decimal places.

$$x \approx 2.397944$$

Finish solving the problem by adding 3 to each side and then dividing each side by 2.

Therefore, the solution to the problem $5^{2x-3} = 18$ is $x \approx 2.397944$.

The directions say, "Take the common logarithm or natural logarithm of each side." You will get the same answer no matter which logarithm you use, it is a matter of personal preference. The only time that you should specifically use a natural logarithm is when dealing with the base e.

Here is another example, solve $8^{4x+1} = 205$.

$$8^{4x+1} = 205$$

Determine if 8 and 205 can be written using the same base. In this case 8 and 205 cannot be written using the same base, so we must use logarithms.

$$\log(8^{4x+1}) = \log(205)$$

Take the common logarithm or natural logarithm of each side.

$$(4x + 1)(\log 8) = \log 205$$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$$4x + 1 = \frac{\log 205}{\log 8}$$

Divide each side by $\log 8$.

$$4x + 1 \approx 2.559827$$

Use a calculator to find $\log 205$ divided by $\log 8$. Round the answer as appropriate, these answers will use 6 decimal places.

$$x \approx 0.389957$$

Finish solving the problem by subtracting 1 from each side and then dividing each side by 4.

Therefore, the solution to the problem $8^{4x+1} = 205$ is $x \approx 0.389957$.

Now that we have looked at a couple of examples of solving exponential equations with different bases, let's list the steps for solving exponential equations that have different bases.

Solving Exponential Equations with Different Bases

Step 1: Determine if the numbers can be written using the same base. If so, stop and use Steps for Solving an Exponential Equation with the Same Base. If not, go to Step 2.

Step 2: Take the common logarithm or natural logarithm of each side.

Step 3: Use the properties of logarithms to rewrite the problem. Specifically, use Property 5 which says $\log_a x^y = y \log_a x$.

Step 4: Divide each side by the logarithm.

Step 5: Use a calculator to find the decimal approximation of the logarithms.

Step 6: Finish solving the problem by isolating the variable.

Examples – Now let's use the steps shown above to work through some examples. These examples will be a mixture of exponential equations with the same base and exponential equations with different bases.

Example 1: Solve $5^{3x+7} = 311$

$$5^{3x+7} = 311$$

Determine if 5 and 311 can be written using the same base. In this case 5 and 311 cannot be written using the same base, so we must use logarithms.

$$\log(5^{3x+7}) = \log(311)$$

Take the common logarithm or natural logarithm of each side.

$$(3x + 7)(\log 5) = \log 311$$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$$3x + 7 = \frac{\log 311}{\log 5}$$

Divide each side by $\log 5$.

$$3x + 7 \approx 3.566334$$

Use a calculator to find $\log 311$ divided by $\log 5$. Round the answer as appropriate, these answers will use 6 decimal places.

$$x \approx -1.144555$$

Finish solving the problem by subtracting 7 from each side and then dividing each side by 3.

Therefore, the solution to the problem $5^{3x+7} = 311$ is $x \approx -1.144555$.

Example 2: Solve $25^{2x-1} = 125^{3x+4}$

$$25^{2x-1} = 125^{3x+4}$$

Determine if 25 and 125 can be written using the same base. In this case both 25 and 125 can be written using the base 5.

$$(5^2)^{2x-1} = (5^3)^{3x+4}$$

Rewrite the problem using the same base.

$$5^{4x-2} = 5^{9x+12}$$

Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.

$$4x - 2 = 9x + 12$$

Since the bases are the same, we can drop the bases and set the exponents equal to each other.

$$x = -\frac{14}{5}$$

Finish solving by adding 2 to each side, subtracting 9x from each side, and then dividing each side by -5.

Therefore, the solution to the problem $25^{2x-1} = 125^{3x+4}$ is $x = -\frac{14}{5}$.

Example 3: Solve $e^{4x-9} = 56$

$$e^{4x-9} = 56$$

Determine if e and 56 can be written using the same base. In this case e and 56 cannot be written using the same base, so we must use logarithms.

$$\ln(e^{4x-9}) = \ln(56)$$

Take the common logarithm or natural logarithm of each side. In this case, we should use the natural logarithm because the base is e.

$$(4x - 9)(\ln e) = \ln 56$$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$$(4x - 9)(1) = \ln 56$$

Use the properties of logarithms to find $\ln e$. Property 2 states that $\ln e = 1$.

$$4x - 9 \approx 4.025352$$

Use a calculator to find $\ln 56$. Round the answer as appropriate, these answers will use 6 decimal places.

$$x \approx 3.256338$$

Finish solving the problem by adding 9 to each side and then dividing each side by 4.

Therefore, the solution to the problem $e^{4x-9} = 56$ is $x \approx 3.256338$.

Example 4: Solve $7^{5x+3} = 512$

$$7^{5x+3} = 312$$

Determine if 7 and 312 can be written using the same base. In this case 7 and 312 cannot be written using the same base, so we must use logarithms.

$$\log(7^{5x+3}) = \log(312)$$

Take the common logarithm or natural logarithm of each side.

$$(5x + 3)(\log 7) = \log 312$$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$$5x + 3 = \frac{\log 312}{\log 7}$$

Divide each side by $\log 7$.

$$5x + 3 \approx 2.951320$$

Use a calculator to find $\log 312$ divided by $\log 7$. Round the answer as appropriate, these answers will use 6 decimal places.

$$x \approx -0.009736$$

Finish solving the problem by subtracting 3 from each side and then dividing each side by 5.

Therefore, the solution to the problem $7^{5x+3} = 312$ is $x \approx -0.009736$.

Example 5: Solve $8^{2x-3} = \left(\frac{1}{16}\right)^{x-2}$

$$8^{2x-3} = \left(\frac{1}{16}\right)^{x-2}$$

Determine if 8 and $1/16$ can be written using the same base. In this case both 8 and $1/16$ can be written using the base 2.

$$(2^3)^{2x-3} = (2^{-4})^{x-2}$$

Rewrite the problem using the same base. Note that $1/16 = 1/2^4 = 2^{-4}$.

$$2^{6x-9} = 2^{-4x+8}$$

Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.

$$6x - 9 = -4x + 8$$

Since the bases are the same, we can drop the bases and set the exponents equal to each other.

$$x = \frac{17}{10}$$

Finish solving by adding 9 to each side, adding $4x$ to each side, and then dividing each side by 10.

Therefore, the solution to the problem $8^{2x-3} = \left(\frac{1}{16}\right)^{x-2}$ is $x = \frac{17}{10}$.

Addition Examples

If you would like to see more examples of solving exponential equations, just click on the link below.

[Additional Examples](#)

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Solve: $4^{3x+5} = 8^{4x-3}$

Problem 2: Solve: $e^{8-3x} = 268$

Problem 3: Solve: $5^{6x+7} = 761$

Problem 4: Solve: $9^{2x+3} = \left(\frac{1}{27}\right)^{3x+1}$

Problem 5: Solve: $3^{2x-5} = 87$

Problem 6: Solve: $4e^{3x-5} = 195$

[Solutions to Practice Problems](#)