Properties of Logarithms – Condensing Logarithms

What are the Properties of Logarithms?

The properties of logarithms are very similar to the properties of exponents because as we have seen before every exponential equation can be written in logarithmic form and vice versa.

Properties for Condensing Logarithms

There are 5 properties that are frequently used for condensing logarithms. These properties are summarized in the table below. When applying the properties of logarithms in the examples shown below and in future examples, the properties will be referred to by number. The condensing of logarithms or writing several logarithms as a single logarithm is often required when solving logarithmic equations.

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<th>Properties for Condensing Logarithms</th>
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<td>Property 1: (0 = \log_a 1) – Zero-Exponent Rule</td>
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<td>Property 2: (1 = \log_a a)</td>
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<td>Property 3: (\log_a x + \log_a y = \log_a (xy)) – Product Rule</td>
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<td>Property 4: (\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)) – Quotient Rule</td>
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<td>Property 5: (y \log_a x = \log_a x^y) – Power Rule</td>
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The 5 properties used for condensing logarithms are the same 5 properties used for expanding logarithms. The only difference if that they are written backwards. For example, Property 1 has been written as \(0 = \log_a 1\) instead of as \(\log_a 1 = 0\).

When we are condensing logarithms there is not a specific order in which these properties must be applied, but some guidelines are listed below.

Guidelines for Condensing Logarithms
- Apply Property 5 and move the number from in front of the logarithm to the exponent of the variable.
- Apply Property 3 or 4 to change the addition and subtraction of the logarithms to multiplication and division.
- Rewrite any rational exponents (fractions) as radicals.
**Examples** – Now let’s use the properties of logarithms to condense logarithms.

**Example 1:** Use the properties of logarithms to write as a single logarithm $5 \log_x x + \frac{2}{3} \log_3 y$.

\[
5 \log_x x + \frac{2}{3} \log_3 y = \log_x x^5 + \log_3 y^{2/3}
\]

Use Property 5 to move the numbers in front of the logarithms to the exponents of the variables.

\[
= \log_x \left(x^5 y^{2/3}\right)
\]

Use Property 3 to change the addition of the logarithms to multiplication.

\[
= \log_x \left(x^{5/3} y^{2/3}\right)
\]

Rewrite the rational exponent (fraction) as a radical.

Thus, $5 \log_x x + \frac{2}{3} \log_3 y = \log_x \left(x^{5/3} y^{2/3}\right)$.

**Example 2:** Use the properties of logarithms to write as a single logarithm $\frac{3}{5} \ln x - 7 \ln y$.

\[
\frac{3}{5} \ln x - 7 \ln y = \ln x^{3/5} - \ln y^7
\]

Use Property 5 to move the numbers in front of the logarithms to the exponents of the variables.

\[
= \ln \left(\frac{x^{3/5}}{y^7}\right)
\]

Use Property 4 to change the subtraction of the logarithms to division.

\[
= \ln \left(\frac{\sqrt[5]{x^3}}{y^7}\right)
\]

Rewrite the rational exponent (fraction) as a radical.

Thus, $\frac{3}{5} \ln x - 7 \ln y = \ln \left(\frac{\sqrt[5]{x^3}}{y^7}\right)$.

**Example 3:** Use the properties of logarithms to write as a single logarithm $5 \log_9 x + 7 \log_y y - 3 \log_z z$.

\[
5 \log_9 x + 7 \log_y y - 3 \log_z z = \log_9 x^5 + \log_y y^7 - \log_z z^3
\]

Use Property 5 to move the numbers in front of the logarithm to the exponents of the variables.

\[
= \log_9 \left(\frac{x^5 y^7}{z^3}\right)
\]

Use Properties 3 and 4 to change the addition of the logarithms to multiplication and the subtraction of the logarithms to division.

Thus, $5 \log_9 x + 7 \log_y y - 3 \log_z z = \log_9 \left(\frac{x^5 y^7}{z^3}\right)$. 
Example 4: Use the properties of logarithms to write as a single logarithm \( \frac{1}{2} \log_2 x - 8 \log_2 y - 5 \log_2 z \).

\[
\frac{1}{2} \log_2 x - 8 \log_2 y - 5 \log_2 z = \log_2 x^{\frac{1}{2}} - \log_2 y^8 - \log_2 z^5
\]

Use Property 5 to move the numbers in front of the logarithm to the exponents of the variables.

\[
= \log_2 \left( \frac{x^{\frac{1}{2}}}{y^8 z^5} \right)
\]

Use Property 3 to change the subtraction of the logarithms to division.

\[
= \log_2 \left( \frac{\sqrt{x}}{y^8 z^5} \right)
\]

Rewrite the rational exponent (fraction) as a radical.

Thus, \( \frac{1}{2} \log_2 x - 8 \log_2 y - 5 \log_2 z = \log_2 \left( \frac{\sqrt{x}}{y^8 z^5} \right) \).

**Note:** In each of the examples shown above, if there was an addition sign in front of the logarithm the variable, \( x, y, \) or \( z \), ended up in the numerator of the final answer. If there was a subtraction or negative sign in front of the logarithm the variable, \( x, y, \) or \( z \), ended up in the denominator of the final answer. So the fastest way to determine the final answer is to look at the sign in front of each logarithm and if the sign in front of the logarithm is addition or positive put the variable in the top of the fraction and if the sign in front of the logarithm is subtraction or negative put the variable in the bottom of the fraction.

Example 5: Use the properties of logarithms to write as a single logarithm \( 4 \log x + 3 \log y - 8 \log z \).

\[
4 \log x + 3 \log y - 8 \log z = \log x^4 + \log y^3 - \log z^8
\]

Use Property 5 to move the numbers in front of the logarithm to the exponents of the variables.

\[
= \log \left( \frac{x^4 y^3}{z^8} \right)
\]

Use Properties 3 and 4 to change the addition of the logarithms to multiplication and the subtraction of the logarithms to division. Notice that for addition the variables, \( x \) and \( y \), ended up in the top part of the fraction and for subtraction the variable, \( z \), ended up in the bottom part of the fraction.

Thus, \( 4 \log x + 3 \log y - 8 \log z = \log \left( \frac{x^4 y^3}{z^8} \right) \).

**Addition Examples**

If you would like to see more examples of condensing logarithm, just click on the link below.

**Additional Examples**
Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Use the properties of logarithms to write as a single logarithm \( \frac{1}{2} \log_4 x - 7 \log_4 y. \)

Problem 2: Use the properties of logarithms to write as a single logarithm \( 3 \log_6 x + \frac{2}{3} \log_6 y - 9 \log_6 z. \)

Problem 3: Use the properties of logarithms to write as a single logarithm \( \frac{1}{5} \log_3 x + \frac{1}{4} \log_3 y. \)

Problem 4: Use the properties of logarithms to write as a single logarithm \( 7 \ln x - 3 \ln y + 8 \ln z. \)

Problem 5: Use the properties of logarithms to write as a single logarithm \( -3 \log_7 x + 6 \log_7 y + 2 \log_7 (z + 3). \)

Problem 6: Use the properties of logarithms to write as a single logarithm \( \frac{2}{7} \log x - 3 \log y - 5 \log z. \)

Solutions to Practice Problems