Using the Change-Of-Base Property to Evaluate Logarithms

Evaluating Logarithms

If we need to evaluate logarithms such as $\log 53$ or $\ln 75$ we can use our calculator to find the approximate value of each by simply typing the problem into our calculator. We can evaluate $\log 53 \approx 1.724276$ and $\ln 75 \approx 4.317488$ o our calculator because calculators can evaluate two types of logarithms, common logarithms (base 10) and natural logarithms (base e). What if we need to evaluate $\log_3 89$ or $\log_7 215$? Neither of these problems can simply get typed into the calculator because calculators do not have log base 3 or log base 7 buttons.

Let's look at how we can evaluate logarithms with bases other than 10 or e. We will start by evaluating the two logarithms above.

Evaluate $\log_3 89$.

$\log_3 89 = x$	Set the problem equal to x because we know it has to equal something.
$89 = 3^x$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side.
$\log(89) = \log(3^x)$	To solve exponential problems with different bases we take the common logarithm or natural logarithm of each side.
$\log 89 = x(\log 3)$	Use the properties of logarithms to rewrite the problem. Use Property 5 to move the exponent out front which turns this into a multiplication problem.
$\frac{\log 89}{\log 3} = x$	Divide each side by log 3.
$4.085733 \approx x$	Use a calculator to find log 89 divided by log 3. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_3 89 = \frac{\log 89}{\log 3} \approx 4.085733.$

The directions say, "Take the common logarithm or natural logarithm of each side." You will get the same answer no matter which logarithm you use, the common logarithm or the natural logarithm.

Evaluate $\log_7 215$.

$\log_7 215 = x$	Set the problem equal to x.
$215 = 7^x$	Rewrite the problem in exponential form by moving the base of the logarithm to the other side.
$\log(215) = \log(7^x)$	To solve exponential problems with different bases we take the common logarithm or natural logarithm of each side.
$\log 215 = x(\log 7)$	Use the properties of logarithms to rewrite the problem. Use Property 5 to move the exponent out front which turns this into a multiplication problem.
$\frac{\log 215}{\log 7} = x$	Divide each side by log 7.
$2.759962 \approx x$	Use a calculator to find log 215 divided by log 7. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_7 215 = \frac{\log 215}{\log 7} \approx 2.759962$.

The Change-Of-Base Property

Notice in each of the examples shown above that final answers were very similar and if we did several more examples the resulting answers would all be very similar. So, to evaluate logarithms with a base other than 10 or e, we can simply rewrite the problem as log of the number divided by the log of the base. We could also rewrite the problem as natural log of the number divided by natural log or the base and the decimal approximates would be the same whether we used common logarithms or natural logarithms.

$\log_3 89 = \frac{\log 89}{\log 3}$ ← log of the number; ← log of the base

$\log_7 215 = \frac{\log 215}{\log 7}$ ← log of the number; ← log of the base

The similarity of these answer lead to the Change-Of-Base Property for evaluating logarithms.

The Change-Of-Base Property – For any logarithmic bases a and b, and any positive number N,

$$\log_b N = \frac{\log_a N}{\log_a b}.$$

The change-of-base property shows that we could use any base a to rewrite the logarithm, but if we want to use our calculator to evaluate the logarithm we need to use base 10 or base e. So, I prefer to write the change-of-base formula as follows.

$$\log_b N = \frac{\log N}{\log b} \text{ or } \log_b N = \frac{\ln N}{\ln b}$$

Examples – Now let's use the steps shown above to work through some examples.

Example 1: Evaluate $\log_9 564$.

$\log_9 564 = \dfrac{\log 564}{\log 9}$ — Use the Change-Of-Base Property to rewrite the problem using common logarithms or natural logarithms.

$\log_9 564 \approx 2.883207$ — Use a calculator to find log 564 divided by log 9. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_9 564 = \dfrac{\log 564}{\log 9} \approx 2.883207$.

Example 2: Evaluate $\log_2 947$.

$\log_2 947 = \dfrac{\log 947}{\log 2}$ — Use the Change-Of-Base Property to rewrite the problem using common logarithms or natural logarithms.

$\log_2 947 \approx 9.887221$ — Use a calculator to find log 947 divided by log 2. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_2 947 = \dfrac{\log 947}{\log 2} \approx 9.887221$.

Example 3: Evaluate $\log_{11} 2164$.

$\log_{11} 2164 = \dfrac{\log 2164}{\log 11}$ — Use the Change-Of-Base Property to rewrite the problem using common logarithms or natural logarithms.

$\log_{11} 2164 \approx 3.202689$ — Use a calculator to find log 2164 divided by log 11. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_{11} 2164 = \dfrac{\log 2164}{\log 11} \approx 3.202689$.

Example 4: Evaluate $\log_6 5534$.

$\log_6 5534 = \dfrac{\log 5534}{\log 6}$ — Use the Change-Of-Base Property to rewrite the problem using common logarithms or natural logarithms.

$\log_6 5534 \approx 4.810169$ — Use a calculator to find log 5534 divided by log 6. Round the answer as appropriate, these answers will use 6 decimal places.

Therefore, $\log_6 5534 = \dfrac{\log 5534}{\log 6} \approx 4.810169$.

Addition Examples

If you would like to see more examples of evaluating logarithms using the change-of-base property, just click on the link below.

Additional Examples

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Evaluate $\log_8 557$

Problem 2: Evaluate $\log_4 99$

Problem 3: Evaluate $\log_{13} 2397$

Problem 4: Evaluate $\log_7 668$

Problem 5: Evaluate $\log_5 8761$

Problem 6: Evaluate $\log_{20} 48147$

Solutions to Practice Problems