

Math 231 CALCULUS 2- Santilli
Series Summary Sheet

test	series	converges	diverges	comments
Nth term Divergence Test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	Just for divergence
Geometric	$\sum_{n=0}^{\infty} ar^n$	$ r < 1$	$ r \geq 1$	Sum: $S = \frac{a}{1-r}$
telescoping	$\sum_{n=1}^{\infty} a_n - a_{(n+1)}$	$\lim_{n \rightarrow \infty} a_n = L$		Sum: $S = a_1 - L$
P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$		Remainder: $ R_n \leq a_{n+1}$
Integral (f is continuous, positive and decreasing)	$\sum_{n=1}^{\infty} f_n$	$\int_1^{\infty} f(n)dn$ converges	$\int_1^{\infty} f(n)dn$ diverges	Remainder: $0 < R_n < \int_n^{\infty} f(x)dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } > 1$	Test fails: $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = 1$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$	Test fails: $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$
Direct Comparison $A_n, B_n > 0$	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ & $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ & $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison $A_n, B_n > 0$	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ & $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$ & $\sum_{n=1}^{\infty} b_n$ diverges	