MAT 241- Calculus 3- Prof. Santilli
Toughloves Chapter 12

1.) Distance between two points in space: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

2.) Equation of a sphere: $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$, where r= radius and (h,k,l) = center.

3.) Midpoint between two points in space: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

4.) Equivalent vectors have the same direction and magnitude.

5.) Vector addition is the long diagonal of the auxiliary parallelogram formed by the vectors.

6.) Vector subtraction is the short diagonal of the auxiliary parallelogram formed by the vectors.

7.) Vectors that are scalar multiples of each other are parallel.

8.) Position vectors always have their initial point at the origin.

9.) Magnitude of a vector: $|\bar{v}| = \sqrt{x^2 + y^2 + z^2}$, where $\bar{v} = \langle x, y, z \rangle$

10.) Base vectors for $\Re^3$ is $\hat{i} = \langle 1,0,0 \rangle$, $\hat{j} = \langle 0,1,0 \rangle$, $\hat{k} = \langle 0.0.1 \rangle$

11.) In 2-D:
Trig form: $\bar{v} = |\bar{v}| \langle \cos\theta, \sin\theta \rangle$, Cartesian form: $\bar{v} = \langle x, y \rangle$, Base vector form: $\bar{v} = x\hat{i} + y\hat{j}$

12.) In 3-D:
Directional Cosine form: $\bar{v} = |\bar{v}| \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$, Cartesian form: $\bar{v} = \langle x, y, z \rangle$, Base vector form: $\bar{v} = x\hat{i} + y\hat{j} + z\hat{k}$

13.) Unit vectors: $\hat{v} = \frac{\bar{v}}{|\bar{v}|} = \frac{\langle x, y, z \rangle}{\sqrt{x^2 + y^2 + z^2}}$ therefore $\bar{v} = |\bar{v}|\hat{v} = v\hat{v} =$(magnitude)(direction)

14.) 3 ways to multiply vectors: scalar multiple, dot (inner) product and cross product.

15.) Dot product: $\bar{v} \bullet \bar{w} = \langle v_1, v_2, v_3 \rangle \bullet \langle w_1, w_2, w_3 \rangle = v_1 w_1 + v_2 w_2 + v_3 w_3$

16.) $\bar{v} \bullet \bar{v} = |\bar{v}|^2$

17.) $\bar{v} \bullet \bar{w} = \bar{w} \bullet \bar{v}$

18.) $\bar{v} \bullet (\bar{w} + \bar{u}) = \bar{v} \bullet \bar{w} + \bar{v} \bullet \bar{u}$

19.) $\bar{0} \bullet \bar{v} = 0$

20.) $(c\bar{v}) \bullet \bar{w} = c(\bar{v} \bullet \bar{w}) = \bar{v} \bullet (c\bar{w})$

21.) $\bar{v} \bullet \bar{w} = |\bar{w}||\bar{v}| \cos\theta$ or $\theta = \arccos\frac{\bar{v} \bullet \bar{w}}{|\bar{w}||\bar{v}|} = \cos^{-1}(\hat{v} \bullet \hat{w})$

22.) If $\bar{w} \bullet \bar{v} = 0$, then $\bar{w} \perp \bar{v}$, the two vectors are orthogonal to each other.

23.) If $\bar{w} \bullet \bar{v} > 0$, then $\theta =$ acute

24.) If $\overline{w} \bullet \overline{v} < 0$, then $\theta =$ obtuse

25.) $\overline{v} \bullet \overline{w} = \pm|\overline{w}||\overline{v}|$, then $\overline{v} \,//\, \overline{w}$

26.) $\overline{P}\text{roj}_{\bar{b}}\, \overline{a} = \dfrac{\overline{a} \bullet \overline{b}}{\overline{b} \bullet \overline{b}}\overline{b} = \left(\dfrac{\overline{a} \bullet \overline{b}}{|\overline{b}|}\right)\hat{b} = (a\cos\theta)\hat{b} = (\overline{a} \bullet \hat{b})\hat{b}$, where $\overline{P}\text{roj}_{\bar{b}}\, \overline{a}$ = vector projection of $\overline{a}$ onto $\overline{b}$ and $Comp_{\bar{b}}\, \overline{a} = \dfrac{\overline{a} \bullet \overline{b}}{|\overline{b}|} = a\cos\theta = \overline{a} \bullet \hat{b}$ is the scalar projection of $\overline{a}$ onto $\overline{b}$, i.e., $\overline{P}_{\bar{b}}\, \overline{a} = (C_{\bar{b}}\, \overline{a})\hat{b}$.

27.) Directional cosines for $\hat{b} = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$.

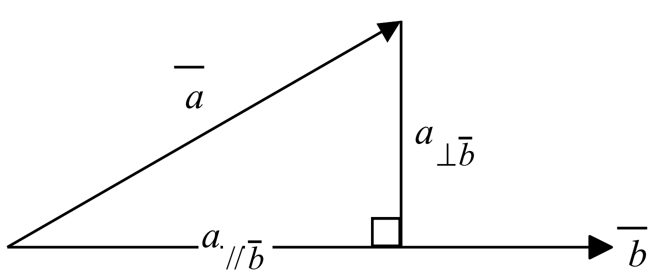
28.) Cross product: $\overline{v} \times \overline{w} = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} =$ vector

29.) $(\overline{a} \times \overline{a}) = \overline{0} = \langle 0,0,0 \rangle$

30.) $(\overline{a} \times \overline{b}) \perp \overline{a}$ and $\overline{b}$ and the direction of $(\overline{a} \times \overline{b})$ is determined by the Right Hand Rule and has a magnitude of $|\overline{a} \times \overline{b}| = |\overline{a}||\overline{b}|\sin\theta$ thus $\theta = \arcsin|\hat{a} \times \hat{b}|$

31.) Distance between a point and a line: $d = \dfrac{|\overrightarrow{QP} \times \overline{v}|}{|\overline{v}|} = |\overrightarrow{QP} \times \hat{v}|$, where $\overline{v}$ is a vector on the line, P is a point on the line and Q is the point off of the line.

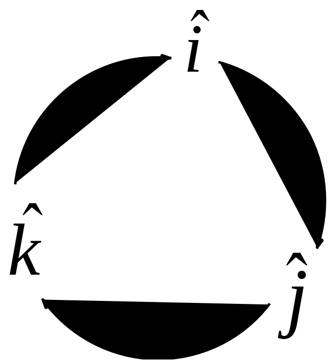
32.) Components of $\overline{a}$ parallel and perpendicular to $\overline{b}$ is $a_{//\bar{b}} = \dfrac{\overline{a} \bullet \overline{b}}{|\overline{b}|} = |\overline{a} \bullet \hat{b}|$, $a_{\perp\bar{b}} = \dfrac{|\overline{a} \times \overline{b}|}{|\overline{b}|} = |\overline{a} \times \hat{b}|$



33.) $(\hat{i}\times\hat{j})=\hat{k}$ $\quad$ $(\hat{j}\times\hat{i})=-\hat{k}$

$(\hat{j}\times\hat{k})=\hat{i}$ and $(\hat{k}\times\hat{j})=-\hat{i}$

$(\hat{k}\times\hat{i})=\hat{j}$ $\quad$ $(\hat{i}\times\hat{k})=-\hat{j}$



34.) $(\bar{a}\times\bar{b})=\bar{0}$, then $\bar{a}\,//\,\bar{b}$

35.) $(\bar{a}\times\bar{b})\neq(\bar{b}\times\bar{a})$ not commutative, whereas $(\bar{a}\times\bar{b})=-(\bar{b}\times\bar{a})$

36.) $(\bar{a}\times\bar{b})\times\bar{c}\neq\bar{a}\times(\bar{b}\times\bar{c})$ not associative in all cases

37.) $(c\bar{a})\times\bar{b}=c(\bar{a}\times\bar{b})=\bar{a}\times(c\bar{b})$

38.) $\bar{a}\times(\bar{b}+\bar{c})=(\bar{a}\times\bar{b})+(\bar{a}\times\bar{c})$

39.) $(\bar{a}+\bar{b})\times\bar{c}=(\bar{a}\times\bar{c})+(\bar{b}\times\bar{c})$

40.) $\bar{a}\bullet(\bar{b}\times\bar{c})=(\bar{a}\times\bar{b})\bullet\bar{c}$ =Triple Scalar Product

41.) $\bar{a}\times\bar{b}\times\bar{c}=(\bar{a}\bullet\bar{c})\bar{b}-(\bar{a}\bullet\bar{b})\bar{c}$ turns a triple cross product into dot products.

42.) Volume of a parallelepiped = $V=|\bar{u}\times\bar{v}\bullet\bar{w}|=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$ = magnitude on the triple scalar product of the 3 vectors that define the sides of the parallelepiped.

43.) Vector equation of line in $\Re^3$: $\langle x-x_o, y-y_o, z-z_o\rangle = t\langle a,b,c\rangle$

44.) Parametric equation of line in $\Re^3$: $\begin{aligned} x &= x_o + at \\ y &= y_o + bt \\ z &= z_o + ct \end{aligned}$

45.) Symmetric equation of line in $\Re^3$: $\dfrac{x-x_o}{a}=\dfrac{y-y_o}{b}=\dfrac{z-z_o}{c}$

46.) Vector equation of plane in $\Re^3$: $\langle x-x_o, y-y_o, z-z_o\rangle \bullet \bar{n}=0$

47.) Cartesian scalar equation of plane in $\Re^3$: $a(x-x_o)+b(y-y_o)+c(z-z_o)=0$ which reduces to the linear equation of plane: $ax+by+cz+d=0$.

48.) Equation of a sphere: $(x-h)^2+(y-k)^2+(z-l)^2=r^2$

49.) Distance between point and a plane in $\Re^3$: $d=\dfrac{\left|\vec{QP}\bullet \bar{n}\right|}{|\bar{n}|}=\left|\vec{QP}\bullet \hat{n}\right|=\dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

50.) In $\Re^3$, x=0 is the y-z plane, y=0 is the x-z plane, and z=0 is the x-y plane.

51.) Cylinder is any surface generated by a series of parallel straight lines (Rulings) moving along a given plane curve (generating curve or directrix)

a.) Right circular cylinder- rulings are perpendicular to the directrix, which traces out a circle.

b.) If the plane curve formed by the directrix is a polygon then the surface is a prism, otherwise, the surface is a cylinder.

52.) Surface of Revolutions revolved about the x-axis: $y^2+z^2=[f(x)]^2$

Surface of Revolutions revolved about the y-axis: $x^2+z^2=[f(y)]^2$

Surface of Revolutions revolved about the z-axis: $x^2+y^2=[f(z)]^2$

53.) Quadric Surfaces general form: (3-D analog of Conic sections)

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$$

54.) Quadric Surfaces Standard form: CENTRAL

$$\pm\frac{(x-h)^2}{a^2}\pm\frac{(y-k)^2}{b^2}\pm\frac{(z-l)^2}{c^2}=1$$

a.) 0 minus = ellipsoid

b.) 1 minus = elliptical hyperbola of one sheet

c.) 2 minus = elliptical hyperbola of two sheets

d.) 3 minus = DNE

e.) 1 minus and =0 = elliptic cone

f.) Axis of Symmetry = odd sign term

55.) Quadric Surfaces Standard form: NON-CENTRAL

$$\pm\frac{(x-h)^2}{a^2}\pm\frac{(y-k)^2}{b^2}=\frac{(z-l)}{c}$$

a.) Same signs = elliptic paraboloid

a.) Different signs = hyperbolic paraboloid (saddle)

56.) Cylindrical coordinates:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

and

$$x^2 + y^2 = r^2 \qquad \arctan\frac{y}{x} = \theta \qquad z = z$$

57.) Spherical coordinates:

$$\rho^2 = x^2 + y^2 + z^2 \qquad \tan\theta = \frac{y}{x} \qquad \cos\varphi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

and

$$x = \rho\sin\varphi\cos\theta \qquad y = \rho\sin\varphi\sin\theta \qquad z = \rho\cos\varphi$$