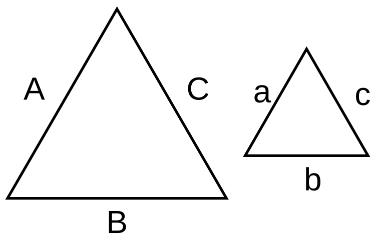
Geometry Review- M.A. Santilli

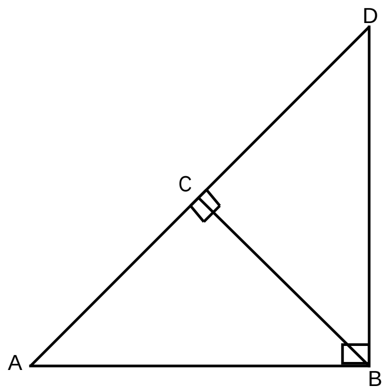
1.) SIMILAR TRIANGLES

a.) Some possible proportions:

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C} \qquad \frac{a}{b} = \frac{A}{B} \qquad \frac{a}{c} = \frac{A}{C} \qquad \frac{c}{b} = \frac{C}{B}$$

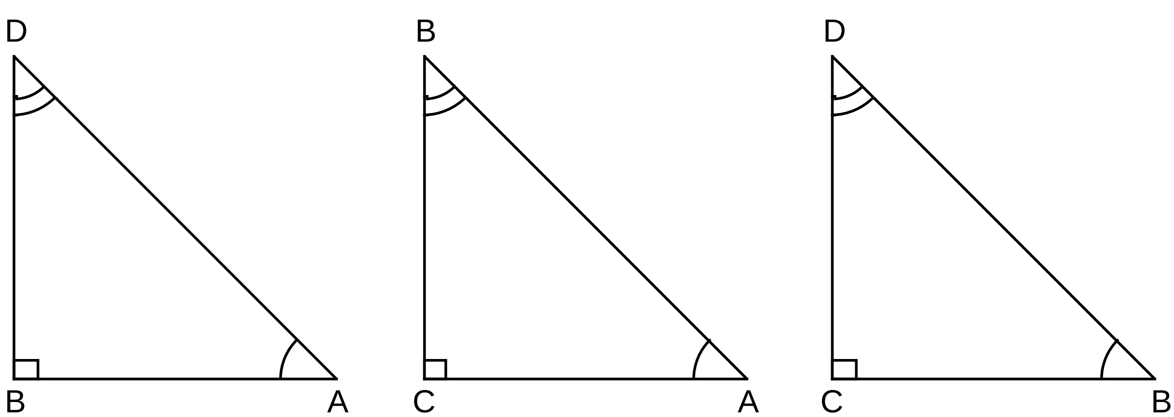


b.) For a right triangle cut by its altitude



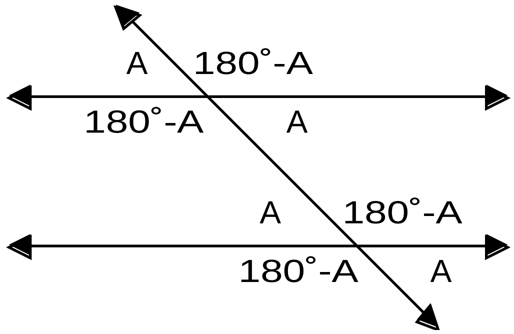
$$\frac{AC}{CB} = \frac{CB}{CD} \qquad \frac{DA}{AB} = \frac{BA}{AC} \qquad \frac{AD}{DB} = \frac{BD}{DC}$$

c.) Or for all possibilities, split into 3 similar triangles:



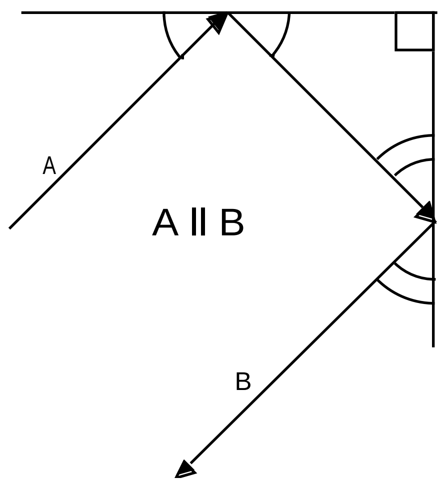
All corresponding sides are proportional.

2.) PARALLEL LINES CUT BY A TRANSVERSAL



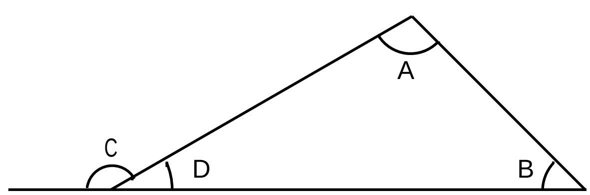
3.) ANGLE OF REFLECTION

For perpendicular mirrors, the incident ray will be parallel to the reflected ray:



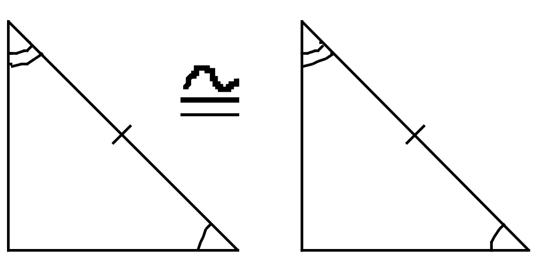
4.) TRIANGLES

a.) $A + B + D = 180^o$ and $C = A + B$

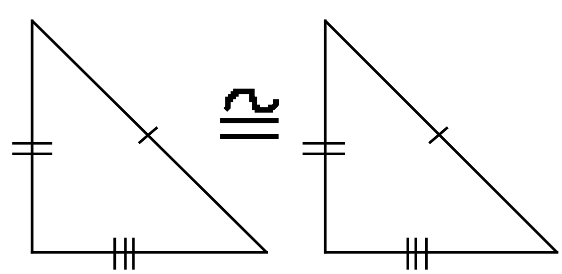


b.) Congruent Triangles:

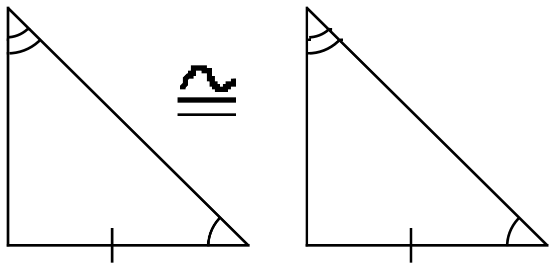
ASA



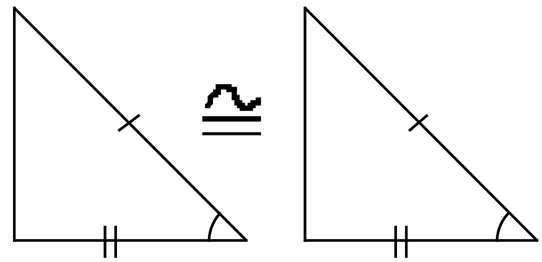
SSS



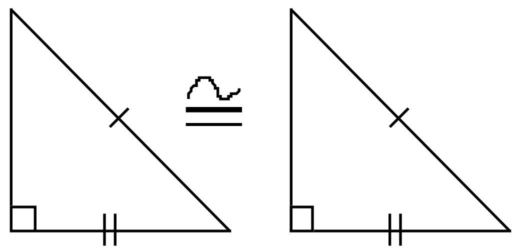
AAS



SAS



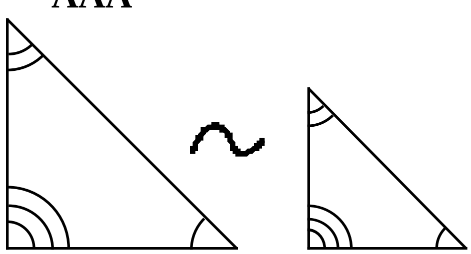
HL



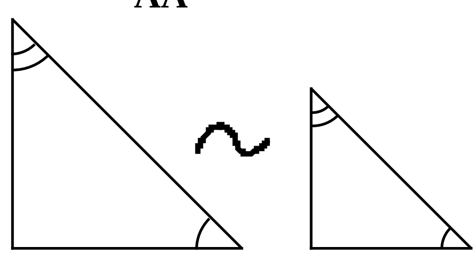
CPCTC- Corresponding Parts of Congruent Triangles are Congruent

c.) Similar Triangles (corresponding sides are therefore proportional):

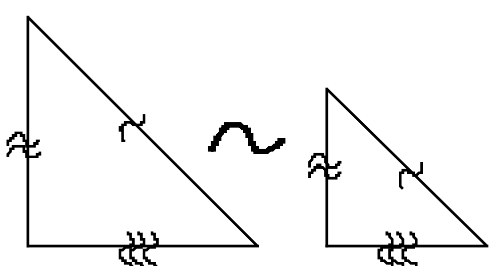
AAA



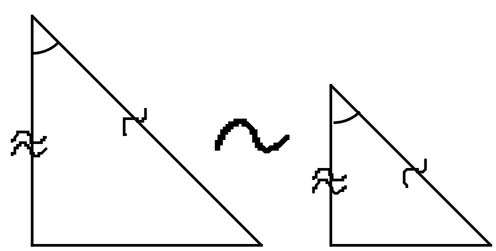
AA



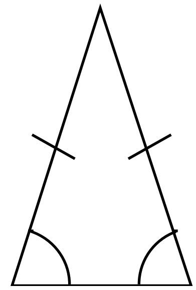
SSS (similar sides)



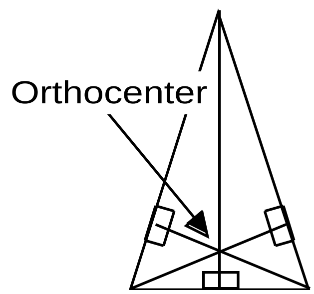
SAS (similar sides)



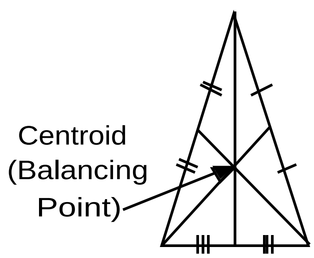
d.) ISOSCELES THEOREM: Base angles are congruent.

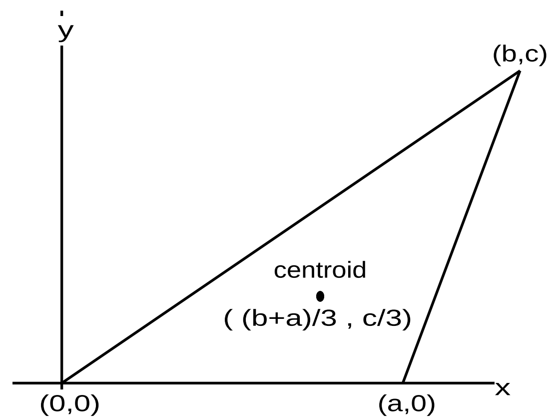


e.) ALTITUDES of a triangle intersect at the ORTHOCENTER:

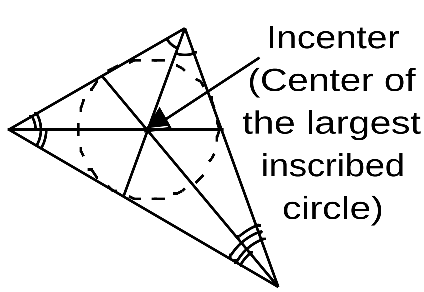


f.) MEDIANS of a triangle intersect at the CENTROID:

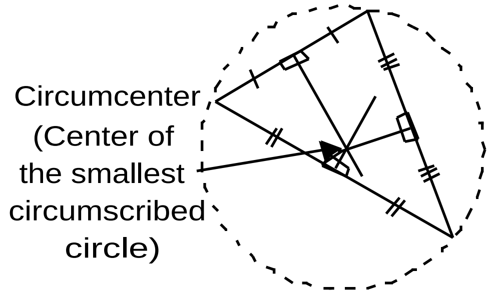




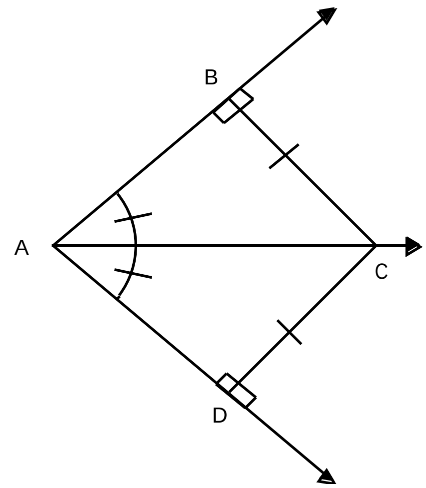
g.) ANGLE BISECTORS of a triangle intersect at the INCENTER:



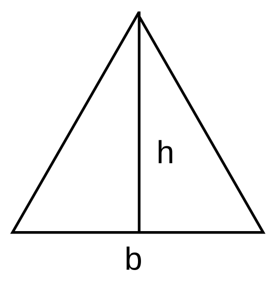
h.) PERPENDICULAR BISECTORS of a triangle intersect at the CIRCUMCENTER:



i.) Any point C on the Angle bisector AC is equidistant to the two sides of the angle, i.e., BC=CD.

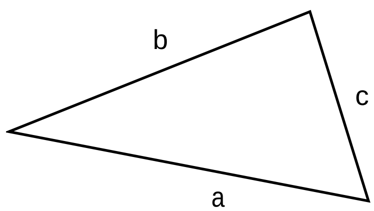


j.) Area of a triangle: A=1/2bh

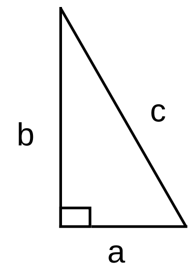


k.) Area of a triangle by the use of only the sides (Heron's Theorem)

$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c) = semiperimeter$$

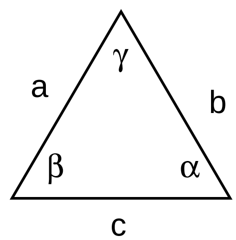


l.) Pythagorean Theorem: $a^2 + b^2 = c^2$ (for right triangles)



m.) Law of the sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

Law of the Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$



5.) POLYGONS (n-gons):

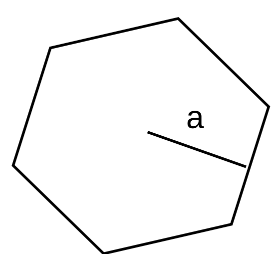
a.) Sum of regular polygon's interior angle $= 180(n-2)^0$ where n= number of sides.

Measurement of each interior angle = $\frac{180(n-2)^0}{n}$

b.) Sum of regular polygon's exterior angles $= 360^o$.

Measurement of each exterior angle = $\frac{360^o}{n}$

c.) Area of polygon: $A = \frac{pa}{2}$ where p= perimeter and a = apothem

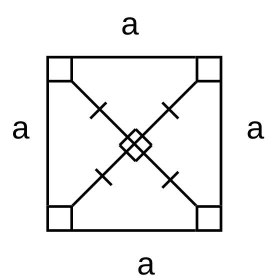


d.) 4-gons (quadrilateral):

1.) Square:

i.) Perimeter: $P = 4a$

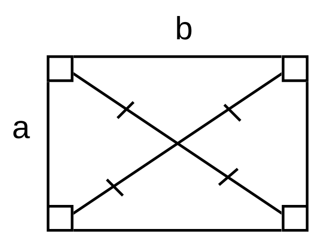
ii.) Area: $A = a^2$



2.) Rectangle:

i.) Perimeter: $P = 2a + 2b$

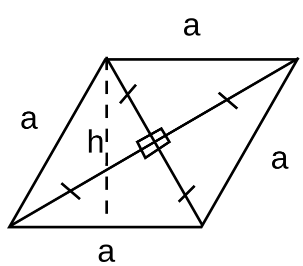
ii.) Area: $A = ab$



3.) Rhombus

i.) Perimeter $P = 4a$

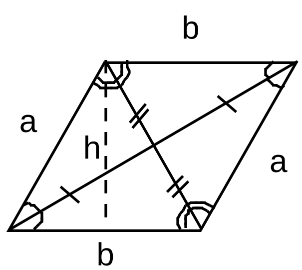
ii.) Area: $A = ah$



4.) parallelogram

i.) Perimeter: $P = 2a + 2b$

ii.) Area: $A = ah$

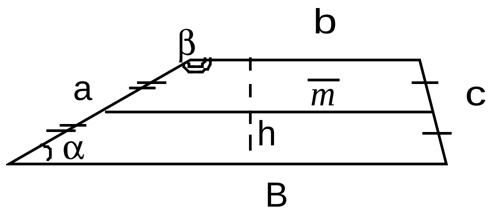


5.) trapezoid

i.) Perimeter: $P = a + c + b + B$

ii.) Area: $A = \frac{h(b+B)}{2} = h\overline{m}$ where $\overline{m}$ is the midsegment

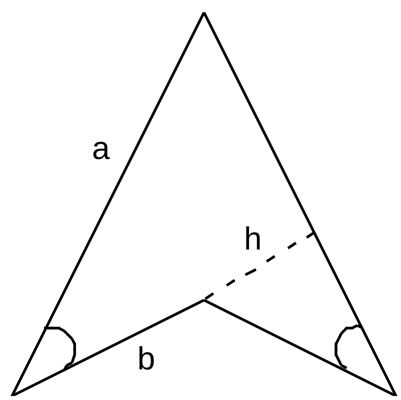
iii.) Adjacent angles are supplementary (i.e., $m\angle\alpha + m\angle\beta = 180^o$)



6.) arrowhead

i.) Perimeter: $P = 2a + 2b$

ii.) Area: $A = ah$

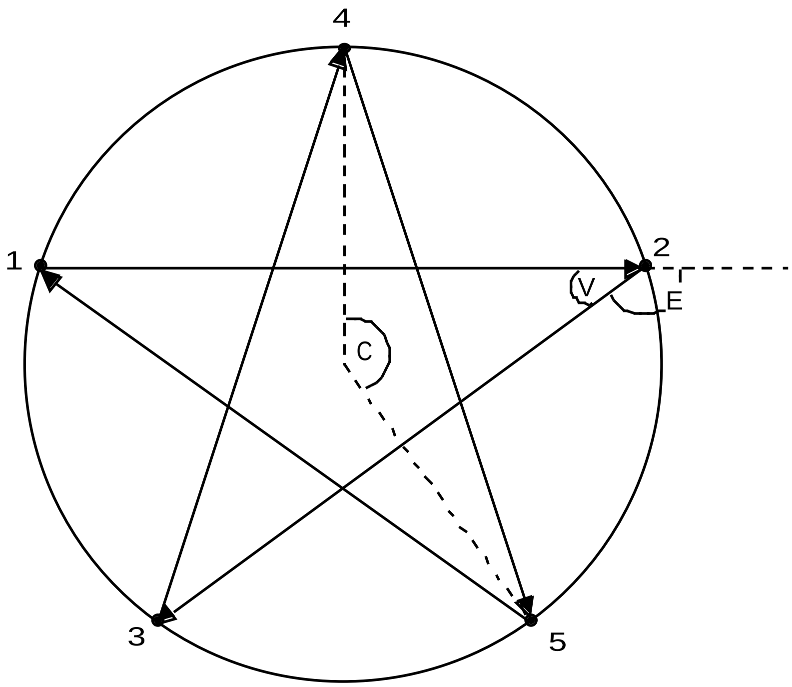


e.) Star Polygons: ($\frac{n}{m} - gons$) where n= total number of vertices and m= number of vertices before a connection.

1.) Measurement of star polygon's vertex angle = $\frac{180\left(\frac{n}{m} - 2\right)^o}{\frac{n}{m}}$

2.) Measurement of star polygon's exterior (turn) angle = $\frac{360^0 m}{n}$ = central angle

3.) example: 5/2gon



5/2 gon = 5 vertices, connecting every other 2 (see numbering above)

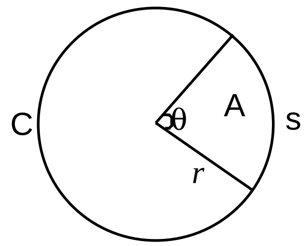
• Measurement of star polygon's vertex angle **V**= $\dfrac{180\left(\frac{5}{2}-2\right)^{o}}{\frac{5}{2}} = 36^{o}$

• Measurement of star polygon's exterior (turn) angle **E**= central angle **C** = $\dfrac{360^{o}(2)}{5} = 144^{o}$

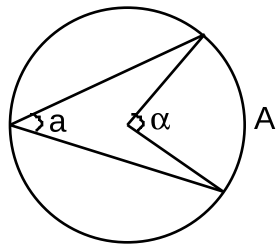
6.) CIRCLES:

a.) Arc length of a sector: $\dfrac{s}{\theta} = \dfrac{C}{2\pi}$ or $\dfrac{s}{\theta} = \dfrac{2\pi r}{2\pi}$ or $s = r\theta$

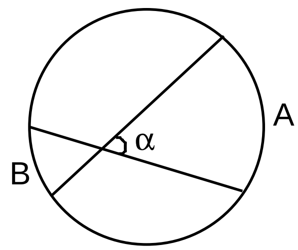
b.) Area of sector: $\dfrac{A}{\theta} = \dfrac{\pi r^2}{2\pi}$ or $A = \dfrac{1}{2} r^2 \theta$



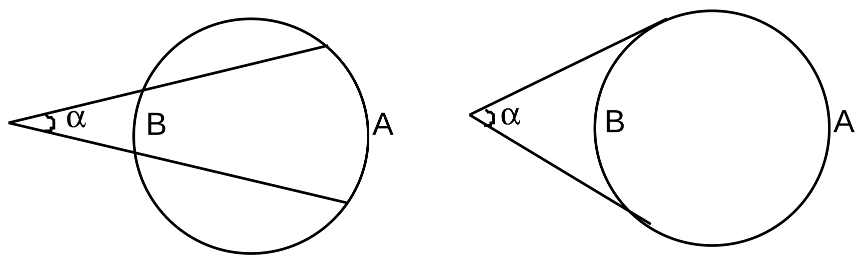
c.) $m\angle\alpha = m\overset{\frown}{A}$ and $m\angle a = \frac{1}{2}m\overset{\frown}{A}$, where $m\overset{\frown}{A}$ is the measurement of the arc A



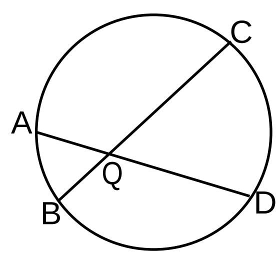
d.) $m\angle\alpha = \frac{1}{2}m\left(\overset{\frown}{A}+\overset{\frown}{B}\right)$



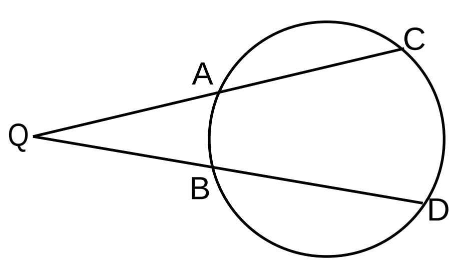
e.) $m\angle\alpha = \frac{1}{2}m\left(\overset{\frown}{A}-\overset{\frown}{B}\right)$



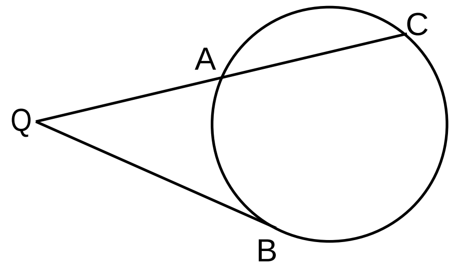
f.) Power of the point Q: $P_Q = (AQ)(QD) = (BQ)(QC)$



g.) Power of the point Q: $P_Q = (QA)(QC) = (QB)(QD)$



h.) Power of the point Q: $P_Q = (QB)^2 = (QA)(QC)$



7.) SIMILAR SOLIDS:

a.) Surface Areas between 2 similar solids: $\frac{S_1}{S_2} = \left(\frac{Edge_1}{Edge_2}\right)^2$

b.) Volumes between 2 similar solids: $\frac{V_1}{V_2} = \left(\frac{Edge_1}{Edge_2}\right)^3$