

MAT 231- Calculus 1 Review- Prof. Santilli
BACKGROUND INFORMATION FOR CALCULUS 2

HYPERBOLIC TRIG FUNCTIONS:

$$1.) \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$2.) \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$3.) \quad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

INVERSE HYPERBOLIC TRIG FUNCTIONS:

$$4.) \quad \sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$5.) \quad \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$6.) \quad \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

LIMITS:

$$1.) \quad \lim_{x \rightarrow a} f(x) = \infty - \infty = \text{indeterminate} .$$

$$2.) \quad \lim_{x \rightarrow a} f(x) = \frac{1}{0} = \pm\infty, DNE .$$

$$3.) \quad \lim_{x \rightarrow a} f(x) = \frac{0}{0} = \text{indeterminate} .$$

$$4.) \lim_{x \rightarrow a} f(x) = \infty^\infty = \infty.$$

$$5.) \lim_{x \rightarrow a} f(x) = 1^\infty = \text{indeterminate}.$$

$$6.) \lim_{x \rightarrow a} f(x) = 0^0 = \text{indeterminate}.$$

$$7.) \lim_{x \rightarrow a} f(x) = \frac{1}{\infty} = 0$$

$$8.) \lim_{x \rightarrow a} f(x) = \frac{\infty}{\infty} = \text{indeterminate}$$

$$9.) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$10.) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$11.) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$12.) \lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1$$

$$13.) \lim_{x \rightarrow \infty} \frac{Ax^{m+1} + \dots}{Bx^m + \dots} = \infty$$

$$14.) \lim_{x \rightarrow \infty} \frac{Ax^m + \dots}{Bx^{m+1} + \dots} = 0$$

$$15.) \lim_{x \rightarrow \infty} \frac{Ax^m + \dots}{Bx^m + \dots} = \frac{A}{B}$$

$$16.) \lim_{x \rightarrow a} f(x) = 0^\infty = 0$$

$$17.) \lim_{x \rightarrow a} f(x) = (0)(\infty) = \text{indeterminate}$$

$$18.) \lim_{x \rightarrow a} f(x) = (-\infty - \infty) = -\infty$$

$$19.) \lim_{x \rightarrow a} f(x) = \infty^{-\infty} = \frac{1}{\infty^{\infty}} = \frac{1}{\infty} = 0$$

$$20.) \lim_{x \rightarrow a} f(x) = \infty^0 = \text{indeterminate}$$

$$21.) \lim_{x \rightarrow a} f(x) = \infty + \infty = \infty$$

$$22.) \lim_{x \rightarrow a} f(x) = \frac{0}{\infty} = 0$$

$$23.) \lim_{x \rightarrow a} f(x) = 0^{-\infty} = \frac{1}{0^{\infty}} = \frac{1}{0} = \pm\infty, DNE$$

DERIVATIVES:

- 1.) The product rule is the first times the derivative to the second plus the second times the derivative of the first, i.e.,

$$\frac{d(f(x)g(x))}{dx} = f(x) \frac{dg(x)}{dx} + g(x) \frac{df(x)}{dx}$$

- 2.) The chain rule is the derivative of the outer function times the derivative of

the inner function, i.e.,
$$\frac{d(f(g(x)))}{dx} = \frac{df}{dg} \frac{dg}{dx}$$

- 3.) The quotient rule is the bottom times the derivative of the top minus the top times the derivative of the bottom all over the bottom squared, i.e.,

$$\frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{[g(x)]^2}$$

4.) $\frac{d(u^a)}{dx} = au^{a-1} \frac{du}{dx}$

5.) $\frac{d(a^u)}{dx} = a^u \ln a \frac{du}{dx}$

6.) $\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$

7.) $\frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx}$

8.) $\frac{d(\log_a u)}{dx} = \frac{1}{u \ln a} \frac{du}{dx}$

9.) $\frac{d|u|}{dx} = \frac{|u|}{u} \frac{du}{dx}$

10.) $\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$

11.) $\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$

12.) $\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$

$$13.) \frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$

$$14.) \frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

$$15.) \frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$$

$$16.) \frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$17.) \frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$18.) \frac{d(\csc^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$19.) \frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \frac{du}{dx}$$

$$20.) \frac{d(\cos^{-1} u)}{dx} = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$21.) \frac{d(\cot^{-1} u)}{dx} = \frac{-1}{1+u^2} \frac{du}{dx}$$

$$22.) \frac{d(\text{constant}^{\text{variable}})}{dx} = (\text{constant}^{\text{variable}}) \ln(\text{constant}) \left(\frac{d(\text{variable})}{dx} \right)$$

$$23.) \frac{d(\text{variable}^{\text{constant}})}{dx} = (\text{constant})(\text{variable}^{\text{constant}-1}) \left(\frac{d(\text{variable})}{dx} \right)$$

$$24.) \frac{d(\text{variable}^{\text{variable}})}{dx} \rightarrow \text{use logarithmic differentiation}$$

$$25.) \frac{d(\text{constant}^{\text{constant}})}{dx} = 0$$

$$26.) \frac{d(\sinh u)}{dx} = \cosh u \frac{du}{dx}$$

$$27.) \frac{d(\cosh u)}{dx} = \sinh u \frac{du}{dx}$$

$$28.) \frac{d(\tanh u)}{dx} = \text{sech}^2 u \frac{du}{dx}$$

$$29.) \frac{d(\coth u)}{dx} = -\text{csch}^2 u \frac{du}{dx}$$

$$30.) \frac{d(\text{sech } u)}{dx} = -\text{sech } u \tanh u \frac{du}{dx}$$

$$31.) \frac{d(\text{csch } u)}{dx} = -\text{csch } u \coth u \frac{du}{dx}$$

$$32.) \frac{d(\sinh^{-1}u)}{dx} = \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx}$$

$$33.) \frac{d(\tanh^{-1}u)}{dx} = \frac{1}{1 - u^2} \frac{du}{dx}$$

$$34.) \frac{d(\cosh^{-1}u)}{dx} = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}$$

$$35.) \frac{d(\coth^{-1}u)}{dx} = \frac{1}{1 - u^2} \frac{du}{dx}$$

$$36.) \frac{d(\operatorname{sech}^{-1}u)}{dx} = \frac{-1}{u\sqrt{1 - u^2}} \frac{du}{dx}$$

$$37.) \frac{d(\operatorname{csch}^{-1}u)}{dx} = \frac{-1}{|u|\sqrt{1 + u^2}} \frac{du}{dx}$$