MAT 231- Calculus 2- Prof. Santilli
Chapter 6 and 8 Toughlove Questions

1.) Volume of a circular disk = $\pi r^2(thickness)$

therefore the differential volume is $dV = \pi r^2 d(thickness)$

2.) Volume of a circular washer = $\pi\left(R^2 - r^2\right)(thickness)$

therefore the differential volume is $dV = \pi\left(R^2 - r^2\right)d(thickness)$

3.) Volume of a cylindrical shell = $2\pi rh(thickness)$

therefore the differential volume is $dV = 2\pi rhd(thickness)$

4.) Volume of a solid = $A_{cross\ section}(thickness)$

therefore the differential volume is $dV = A_{cross\ section}d(thickness)$

5.) Arc length of a curve: $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$, $ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}dy$, $ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt$,

$ds = \sqrt{(dx)^2+(dy)^2}$, $ds = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}d\theta$

6.) Surface area for solid of revolution: $dS = 2\pi rds$

about the x-axis: $dS = 2\pi yds$, about the y-axis: $dS = 2\pi xds$

7.) Average value of a function: $f_{avg} = \dfrac{\int_a^b f(x)dx}{b-a}$

8.) Forces:

Spring Force $f(x) = kx$

Weight $f = mg$

Gravitational $f(r) = \dfrac{k}{r^2}$

9.) $dW = (distance)dF = (distance)gdm = (distance)g\rho dV = (distance)g\rho Ad(thickness)$

10.) Mass density of water $= \rho = 1000\dfrac{kg}{m^3}$, Weight density of water $= \rho g = 62.5\dfrac{lbs}{ft^3}$.

11.) Pascal's Principal- Any point in a liquid, the hydrostatic pressure on an object is the same in all directions at any depth h.

12.) $dF_{hydrostatic} = gdm = g\rho dV = g\rho(depth)dA_{\perp} = g\rho(depth)f(slice)d(thickness)$

13.) Center of Gravity:

$$\bar{x}_g = \frac{\tau_y}{F} = \frac{\int_a^b \rho(x)g(x)xf(x)dx}{\int_a^b \rho(x)g(x)f(x)dx} \qquad \bar{y}_g = \frac{\tau_x}{F} = \frac{\frac{1}{2}\int_a^b \rho(x)g(x)[f(x)]^2 dx}{\int_a^b \rho(x)g(x)f(x)dx}$$

14.) Center of Mass, (g= constant):

$$\bar{x}_m = \frac{M_y}{m} = \frac{\int_a^b \rho(x)xf(x)dx}{\int_a^b \rho(x)f(x)dx} \qquad \bar{y}_m = \frac{M_x}{m} = \frac{\frac{1}{2}\int_a^b \rho(x)[f(x)]^2 dx}{\int_a^b \rho(x)f(x)dx}$$

15.) Centroid, (g and ρ=constant):

$$\bar{x}_c = \frac{G_y}{A} = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx} \qquad \bar{y}_c = \frac{G_x}{A} = \frac{\frac{1}{2}\int_a^b [f(x)]^2 dx}{\int_a^b f(x)dx}$$

16.) Centroid for a region that lies between 2 curves, (g and ρ=constant):

$$\bar{x}_c = \frac{G_y}{A} = \frac{\int_a^b x[f(x)-g(x)]dx}{\int_a^b [f(x)-g(x)]dx} \qquad \bar{y}_c = \frac{G_x}{A} = \frac{\frac{1}{2}\int_a^b \left([f(x)]^2 - [g(x)]^2\right)dx}{\int_a^b [f(x)-g(x)]dx}$$

Where τ_y, τ_x = Torque (force moments), M_y, M_x = Moments (mass moments) G_y, G_x = Geometric Moments, F = Total Force, m = Total Mass, and A = Total Area.