MAT 231- Calculus 2- Prof. Santilli
Review Toughloves

HYPERBOLIC TRIG FUNCTIONS:

1.) $\sinh x = \dfrac{e^x - e^{-x}}{2}$	2.) $\operatorname{csch} x = \dfrac{2}{e^x - e^{-x}}$
3.) $\cosh x = \dfrac{e^x + e^{-x}}{2}$	4.) $\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$
5.) $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	6.) $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

INVERSE HYPERBOLIC TRIG FUNCTIONS:

1.) $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \quad x \in (-\infty, +\infty)$	2.) $\operatorname{csch}^{-1} x = \ln\left(\dfrac{1}{x} + \dfrac{\sqrt{x^2+1}}{\lvert x\rvert}\right), \quad x \neq 0$
3.) $\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right), \quad x \in [1, \infty)$	4.) $\operatorname{sech}^{-1} x = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right), \quad x \in (0,1]$
5.) $\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right), \quad \lvert x\rvert < 1$	6.) $\coth^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right) \quad \lvert x\rvert > 1$

LIMITS:

1.) $\lim_{x\to a} f(x) = \infty - \infty = \text{ indeterminate}$.

2.) $\lim_{x\to a} f(x) = \frac{1}{0} = \pm\infty, DNE$.

3.) $\lim_{x\to a} f(x) = \frac{0}{0} = \text{indeterminate}$.

4.) $\lim_{x\to a} f(x) = \infty^{\infty} = \infty$.

5.) $\lim_{x\to a} f(x) = 1^{\infty} = \text{indeterminate}$.

6.) $\lim_{x\to a} f(x) = 0^{0} = \text{ indeterminate}$.

7.) $\lim_{x\to a} f(x) = \frac{1}{\infty} = 0$

8.) $\lim_{x\to a} f(x) = \frac{\infty}{\infty} = \text{indeterminate}$

9.) $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

10.) $\lim_{x\to 0}\frac{\sin x}{x} = 1$

11.) $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

12.) $\lim_{x\to 0}\left(\frac{e^x - 1}{x}\right) = 1$

13.) $\lim_{x\to\infty} \frac{Ax^n + ...}{Bx^m + ...} = \infty, \quad n > m$

14.) $\lim_{x\to\infty} \frac{Ax^n + ...}{Bx^m + ...} = 0, \quad n < m$

15.) $\lim_{x\to\infty} \frac{Ax^m + ...}{Bx^m + ...} = \frac{A}{B}$

16.) $\lim_{x\to a} f(x) = 0^{\infty} = 0$

17.) $\lim_{x\to a} f(x) = (0)(\infty)$ = indeterminate

18.) $\lim_{x\to a} f(x) = (-\infty - \infty) = -\infty$

19.) $\lim_{x\to a} f(x) = \infty^{-\infty} = \frac{1}{\infty^{\infty}} = \frac{1}{\infty} = 0$

20.) $\lim_{x\to a} f(x) = \infty^{0}$ = indeterminate

21.) $\lim_{x\to a} f(x) = \infty + \infty = \infty$

22.) $\lim_{x\to a} f(x) = \frac{0}{\infty} = 0$

23.) $\lim_{x\to a} f(x) = \frac{\infty}{0} = \pm\infty, DNE$

24.) $\lim_{x\to a} f(x) = 0^{-\infty} = \frac{1}{0^{\infty}} = \frac{1}{0} = \pm\infty, DNE$

DERIVATIVES:

1.)The product rule is the first times the derivative to the second plus the second times the derivative of the first, i.e.,

$$\frac{d\left(\mathrm{f(x)g(x)}\right)}{dx} = f(x)\frac{dg(x)}{dx} + g(x)\frac{df(x)}{dx}$$

2.)The chain rule is the derivative of the outer function times the derivative of the inner function, i.e., $\frac{d\left(\mathrm{f(g(x))}\right)}{dx} = \frac{df}{dg}\frac{dg}{dx}$

3.)The quotient rule is the bottom times the derivative of the top minus the top times the derivative of the bottom all over the bottom squared, i.e.,

$$\frac{d\left(\frac{\mathrm{f(x)}}{\mathrm{g(x)}}\right)}{dx} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{[g(x)]^2}$$

4.) $\frac{d\left(\mathrm{u}^{a}\right)}{dx} = a\mathrm{u}^{a-1}\frac{du}{dx}$

5.) $\frac{d\left(\mathrm{a}^{u}\right)}{dx} = \mathrm{a}^{u}\ln a\frac{du}{dx}$

6.) $\frac{d\left(\mathrm{e}^{u}\right)}{dx} = \mathrm{e}^{u}\frac{du}{dx}$

7.) $\frac{d\left(\ln \mathrm{u}\right)}{dx} = \frac{1}{u}\frac{du}{dx}$

8.) $\frac{d\left(\log_{a}\mathrm{u}\right)}{dx} = \frac{1}{u\ln a}\frac{du}{dx}$

9.) $\frac{d|u|}{dx} = \frac{|u|}{u}\frac{du}{dx}$

10.) $\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$

11.) $\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$

12.) $\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$

13.) $\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$

14.) $\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$

15.) $\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$

16.) $\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

17.) $\frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

18.) $\frac{d(\csc^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

19.) $$\frac{d\left(\tan^{-1}u\right)}{dx}=\frac{1}{1+u^2}\frac{du}{dx}$$

20.) $$\frac{d\left(\cos^{-1}u\right)}{dx}=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

21.) $$\frac{d\left(\cot^{-1}u\right)}{dx}=\frac{-1}{1+u^2}\frac{du}{dx}$$

22.) $$\frac{d\left(\text{constant}^{\text{variable}}\right)}{dx}=\left(\text{constant}^{\text{variable}}\right)\ln(\text{constant})\left(\frac{\text{d(variable)}}{\text{dx}}\right)$$

23.) $$\frac{d\left(\text{variable}^{\text{constant}}\right)}{dx}=(\text{constant})\left(\text{variable}^{\text{constant}-1}\right)\left(\frac{\text{d(variable)}}{\text{dx}}\right)$$

24.) $$\frac{d\left(\text{variable}^{\text{variable}}\right)}{dx}\rightarrow \text{use logarithmic differentiation}$$

25.) $$\frac{d\left(\text{constant}^{\text{constant}}\right)}{dx}=0$$

26.) $$\frac{d(\sinh u)}{dx}=\cosh u\frac{du}{dx}$$

27.) $$\frac{d(\cosh u)}{dx}=\sinh u\frac{du}{dx}$$

28.) $\frac{d(\tanh u)}{dx} = \operatorname{sech}^2 u \frac{du}{dx}$

29.) $\frac{d(\coth u)}{dx} = -\operatorname{csch}^2 u \frac{du}{dx}$

30.) $\frac{d(\operatorname{sech} u)}{dx} = -\operatorname{sech} u \tanh u \frac{du}{dx}$

31.) $\frac{d(\operatorname{csch} u)}{dx} = -\operatorname{csch} u \coth u \frac{du}{dx}$

32.) $\frac{d(\sinh^{-1} u)}{dx} = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$

33.) $\frac{d(\tanh^{-1} u)}{dx} = \frac{1}{1-u^2} \frac{du}{dx}$

34.) $\frac{d(\cosh^{-1} u)}{dx} = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}$

35.) $\frac{d(\coth^{-1} u)}{dx} = \frac{1}{1-u^2} \frac{du}{dx}$

36.) $\frac{d(\operatorname{sech}^{-1} u)}{dx} = \frac{-1}{u\sqrt{1-u^2}} \frac{du}{dx}$

37.) $\frac{d(\operatorname{csch}^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{1+u^2}} \frac{du}{dx}$

INTEGRALS:

Table of Fundamental Integration Formulas
(TOUGHLOVE QUESTIONS)

1.) $\int K dx = Kx + c$

2.) $\int u^n du = \frac{u^{n+1}}{n+1} + c$ where n ≠ -1

3.) $\int \frac{|u|}{u} du = |u| + c$

4.) $\int e^u du = e^u + c$

5.) $\int a^u du = \frac{a^u}{\ln a} + c$

6.) $\int \frac{du}{u} = \ln u + c$

7.) $\int \sin u du = -\cos u + c$

8.) $\int \sinh u du = \cosh u + c$

9.) $\int \cos u du = \sin u + c$

10.) $\int \cosh u du = \sinh u + c$

11.) $\int \sec^2 u du = \tan u + c$

12.) $\int \text{sech}^2 u du = \tanh u + c$

13.) $\int \csc^2 u du = -\cot u + c$

14.) $\int \text{csch}^2 u du = -\coth u + c$

15.) $\int \sec u \tan u du = \sec u + c$

16.) $\int \text{sech } u \tanh u du = -\text{sech } u + c$

17.) $\int \csc u \cot u du = -\csc u + c$

18.) $\int \text{csch } u \coth u du = -\text{csch } u + c$

19.) $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c$

20.) $\int \frac{du}{\sqrt{1+u^2}} = \sinh^{-1} u + c = \ln\left(u + \sqrt{u^2+1}\right) + c$

21.) $\int \frac{du}{1+u^2} = \tan^{-1} u + c$

22.) $\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + c = \ln\left(u + \sqrt{u^2-1}\right) + c$

23.) $\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + c$

24.) $\int \frac{du}{u\sqrt{u^2+1}} = -\text{csch}^{-1}|u| + c$

25.) $\int \frac{du}{u\sqrt{1-u^2}} = -\sec h^{-1} u + c$

26.) $\int \frac{du}{1-u^2} = \begin{cases} \tanh^{-1} u + c & \text{if } |u| < 1 \\ \coth^{-1} u + c & \text{if } |u| > 1 \end{cases} = \frac{1}{2} \ln\left|\frac{1+u}{1-u}\right| + c$ if $u \neq 1$

27.) $\int |u| du = \frac{|u|u}{2} + c$

28.)

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1.) $\int_{-a}^{+a} f_{even}\,dx = 2\int_{0}^{+a} f_{even}\,dx$

2.) $\int_{-a}^{+a} f_{odd}\,dx = 0$

3.) $\int_{a}^{b} f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x =$ Infinite sum

4.) $\int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx$

5.) Integration Sweep Preference #1: Minimize "BROOM'S" variable ends.

6.) Integration Sweep Preference #2: Do not let "BROOM" jump functions.

7.) $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f(h(x))\frac{dh}{dx} - f(g(x))\frac{dg}{dx}$

8.) $\int_{a}^{b} f(x)dx = F(b) - F(a)$

9.) $\int_{t_1}^{t_2} v(t)dt = s(t_2) - s(t_1) = displacement$

10.) $A_{trapezoid} = \frac{1}{2}h(b_1 + b_2)$

11.) $A_{sector} = \frac{1}{2}r^2\theta_{radians}$

12.) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

13.) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

14.) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

15.) $\sum_{i=1}^{n} C = Cn$

16.) Mathematical Induction:
- a.) True for first term.
- b.) Assume true for Kth term.
- c.) Prove true for (K+1)th term

Therefore true for all terms.

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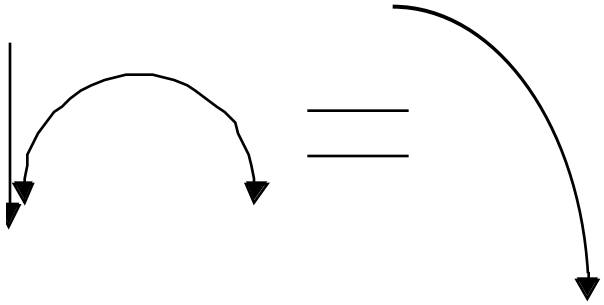
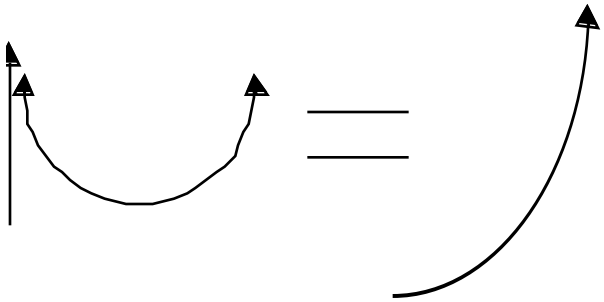
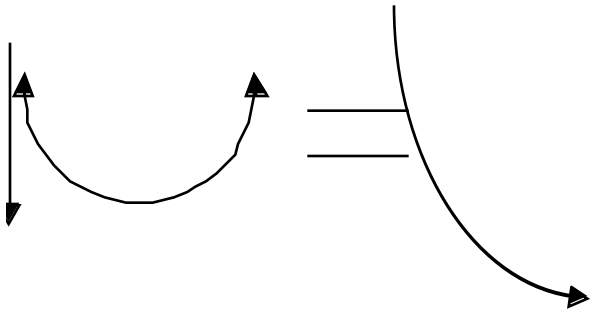
Tendencies and Concavity



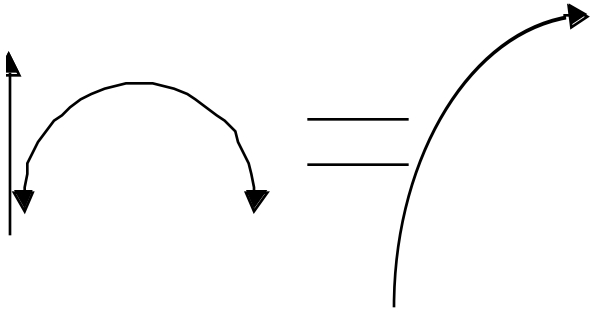
f'(x)= (-)
f''(x)= (-)



f'(x)= (+)
f''(x)= (+)



f'(x)= (-)
f''(x)= (+)



f'(x)= (+)
f''(x)= (-)

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1.) Critical Points can be local maxima, local minima, or neither.

2.) The function is increasing when the first derivative is positive.

3.) The function is concave up when the second derivative is positive.

4.) The function is decreasing when the first derivative is negative.

5.) The function is concave down when the second derivative is negative.

6.) Inflection points are points on the function where the curve changes concavity.

7.) Inflection points are found when the second derivative is zero or does not exist, i.e., $f''(x) = 0$ or DNE

8.) Critical points are points on the function where the curve changes tendency.

9.) Critical points are found when the first derivative is zero or does not exist, i.e., $f'(x) = 0$ or DNE

10.) An absolute extrema can be found at either the function's end points or at its CP's.

11.) A function can be maximized by setting f'(x)= 0, DNE. The critical points, along with the end points are then tested for optimization.

12.) The two types of integrals are indefinite and definite.

13.) If f(x) is discontinuous at a point that falls between the limits of a definite integral, then the integral does not exist, i.e., $\int_a^b f(x)dx = DNE$ if $a \le c \le b$ and $f(c) = DNE$.

14.) Vertical asymptotes act like Critical points or Points of Inflection.

15.) L'Hospital's rule can only be used for limits that tend towards the indeterminate forms of $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.

16.) Newton's Method is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

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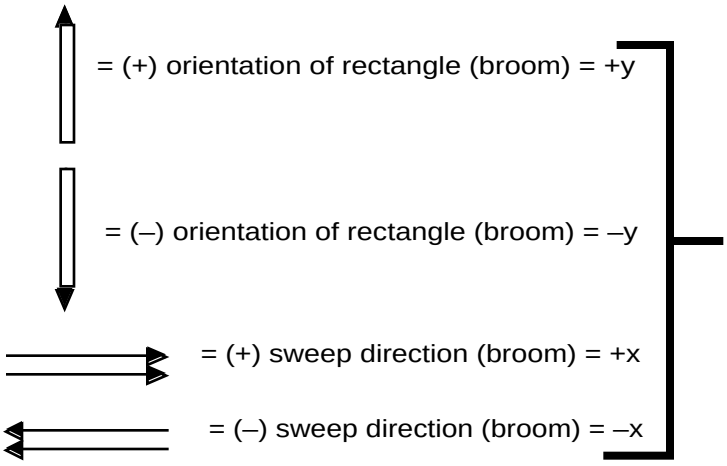
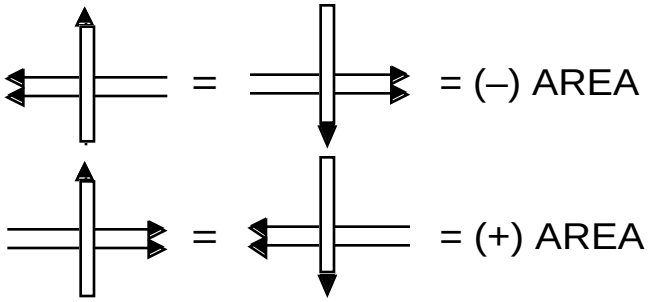
Positive or Negitive Areas

For dx





For dy