

**Math 231 CALCULUS 2- Santilli**  
**Worksheet A –Miscellaneous Integration Substitutions.**

$$1.) \quad \int \sec x dx = \int \frac{dx}{\cos x} = \ln \left| \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right| + C$$

$$2.) \quad \int \frac{dx}{\sqrt{x^2 - 6x + 13}} = \ln(x - 3 + \sqrt{x^2 - 6x + 13}) + C$$

$$= \sinh^{-1}\left(\frac{x-3}{2}\right) + C$$

$$3.) \quad \int \frac{dx}{\sqrt{4+x^2}} = \ln \frac{1}{2}(x + \sqrt{4+x^2}) + C = \sinh^{-1}\left(\frac{x}{2}\right) + C$$

$$4.) \quad \int \frac{xdx}{\sqrt{x^4-1}} = \frac{1}{2} \ln(x^2 + \sqrt{x^4-1}) + C = \frac{1}{2} \cosh^{-1} x^2 + C$$

$$5.) \quad \int \frac{\cos x dx}{\sqrt{4 - \cos^2 x}} = \ln(\sin x + \sqrt{4 - \cos^2 x}) + C$$

$$= \sinh^{-1}\left(\frac{\sin x}{\sqrt{3}}\right) + C$$

$$6.) \quad \int \frac{dx}{5 + 4 \cos x} = \frac{2}{3} \arctan\left(\frac{1}{3} \tan \frac{x}{2}\right) + C$$

$$7.) \quad \int \frac{dx}{\cos x - \sin x + 1} = -\ln \left| 1 - \tan \frac{x}{2} \right| + C$$

$$8.) \quad \int \frac{dx}{\sin x + \tan x} = \frac{1}{2} \ln \left| \tan \frac{x}{2} \right| - \frac{1}{4} \tan^2 \frac{x}{2} + C$$

$$9.) \quad \int_0^{\frac{\pi}{2}} \frac{dx}{5 \sin x + 3} = \frac{\ln 3}{4}$$

$$10.) \quad \int x^5 \sqrt{x^2 + 4} dx = \frac{1}{105} (x^2 + 4)^{\frac{3}{2}} (15x^4 - 48x^2 + 128) + C$$

$$11.) \quad \int \frac{xdx}{3 + \sqrt{x}} = \frac{2}{3} x^{\frac{3}{2}} - 3x + 18\sqrt{x} - 54 \ln(3 + \sqrt{x}) + C$$

$$12.) \quad \int \frac{dx}{x\sqrt{1+4x}} = \ln \left| \frac{\sqrt{1+4x}-1}{\sqrt{1+4x}+1} \right| + C$$

$$13.) \quad \int \frac{dx}{1 + \sqrt[3]{x-2}}$$

$$= \frac{3}{2} (x-2)^{\frac{2}{3}} - 3(x-2)^{\frac{1}{3}} + \ln \left| 1 + (x-2)^{\frac{1}{3}} \right| + C$$

$$14.) \quad \int_{\frac{1}{2}}^2 \frac{dx}{\sqrt{2x}(\sqrt{2x}+9)} = \ln \left( \frac{11}{10} \right)$$

$$15.) \quad \int \frac{dx}{\sqrt{2x} - \sqrt{x+4}} = 2\sqrt{2x} + 2\sqrt{x+4}$$

$$+ 4\sqrt{2} \ln \left| \frac{(\sqrt{2x} - \sqrt{x+4})(\sqrt{x} - 2)}{x-4} \right| + C$$