FUNCTIONS
A function is a relationship between two or more variables. For every input to a function, there is always only one output.

\[ x \rightarrow f(x) \rightarrow y \]

**Components of a function:**
\[ y = f(x) = 5x + 2. \]

\( y \): "dependent variable" or "range"
\( x \): "independent variable" or "domain"

**Example**

Is \( y = f(x) \) true?
Yes, because there is only one \( y \) value for any given \( x \).

Is \( x = f(y) \) true?
No, because there are multiple \( x \) values possible for some \( y \) values.

**Notation**
\[ g(x) = 3x^2 \] is the same as \[ y = 3x^2 \]
\[ g(2) = 3x^2 \] is the same as \[ y = 3x^2 \] when \( x = 2 \).
Consider \[ f(x) = 5x + 2 \]
\[ g(x) = 3x^2 \]

What is \( f(x) \cdot g(x) \)?
\[ (5x + 2)(3x^2) \]
\[ \frac{f(x)}{g(x)} \]?
\[ \frac{5x + 2}{3x^2} \]
\[ f \circ g \]?
\[ f(g(x)) = 5(3x^2) + 2 \]

The "\( \circ \)" symbol indicates a composite function.

**PIECEWISE FUNCTION**

A piecewise function is one comprised of two or more pieces of functions.

**Example**

\[ f(x) = \begin{cases} 
3x^2 & \text{when } x > 0 \\
5x - 2 & \text{when } x \leq 0 
\end{cases} \]

So
\[ f(2) = 12 \text{ and } f(-3) = -17 \]

**Example**

\[ f(x) = \begin{cases} 
3x^2 & \text{when } x < 0 \\
x & \text{when } -1 < x < 1 \\
x + 2 & \text{when } x \geq 1 
\end{cases} \]

The gaps in the piecewise graphs are "discontinuity."

So the piecewise function looks like

so the piecewise function looks like

\[ f(x) = y \]
\[ f(y) = x \]
\[ f(x) = y \text{ is true} \]
\[ f(y) = x \text{ is true} \]
Absolute Value —

An absolute value is a piecewise function which can be written as

\[ y = |x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases} \]

An absolute value is the distance of a value on the number line from zero. The distance of a value in a direction is its "displacement."

Taking the absolute value of a negative number is the same as multiplying the number by \(-1\).

**Example**

\[ |-x| = (-x)(-1) = x \]

**Example**

Consider \( y = |x^2 - 4| \)

\[ = \begin{cases} (x^2 - 4) & \text{when } (x^2 - 4) \geq 0 \\ -(x^2 - 4) & \text{when } (x^2 - 4) < 0 \end{cases} \]

\[ = \begin{cases} x^2 - 4 & \text{when } (x^2 - 4) \geq 0 \\ 4 - x^2 & \text{when } (x^2 - 4) < 0 \end{cases} \]

...to be cont'd

An alternate form for \(|x|\) is \(\sqrt{x^2}\). Remember, \(2 = \sqrt{4}\) and \(-2 = -\sqrt{4}\) but \(\pm 2 = \sqrt{4}\).
example: \[ |x-3| = 7 \implies x = -10 \]
\[ (x-3)^2 = 7^2 \implies x = 4 \]
\[ x^2 - 6x + 9 = 49 \implies x + 10 = 0 \]
\[ x^2 - 6x + 46 = 0 \implies x - 4 = 0 \]
\[ (x+10)(x-4) = 0 \]

EVEN & ODD FUNCTIONS

If all the \( x \) values in a function are changed to \(-x\) and the result is the original function, the function is \underline{even}. That is, if \( f(x) = f(-x) \) then \( f(x) \) is even.

\[
\begin{align*}
\text{example} & \quad y = x^4 + x^2 \\
& \quad y = (-x)^4 + (-x)^2 \\
& \quad y = x^4 + x^2 \quad \text{EVEN}
\end{align*}
\]

If all the \( x \) values in a function are changed to \(-x\) and the result is the negative of the original function, the function is \underline{odd}. That is, if \( f(x) = -f(x) \) then \( f(x) \) is odd.

\[
\begin{align*}
\text{example} & \quad y = x^3 \\
& \quad y = (-x)^3 \\
& \quad y = -x^3 \quad \text{ODD}
\end{align*}
\]

\[
\begin{align*}
\text{example} & \quad y = x^3 + 5 \\
& \quad y = (-x)^3 + 5 \\
& \quad y = -x^3 + 5 \quad \text{N\O\T ODD (also not even)}
\end{align*}
\]

May 31-4
SYMMETRY

A function has \( x \)-symmetry if, by replacing all \( y \)'s with negative \( y \)'s, the function reduces to its original form.

**TEST:** \( y \rightarrow -y \rightarrow y \)

**Example:** \( x = y^2 \rightarrow x = (-y)^2 = y^2 \)

A function has \( y \)-symmetry if, by replacing all \( x \)'s with negative \( x \)'s, the function reduces to its original form.

**TEST:** \( x \rightarrow -x \rightarrow x \)

**Example:** \( y = x^2 \rightarrow y = (-x)^2 = x^2 \)

**NOTE:** All \( y \)-symmetrical functions are even. The test is the same.

A function has origin symmetry if, by replacing all \( x \)'s with negative \( x \)'s and \( y \)'s with negative \( y \)'s, the function reduces to its original form.

**TEST:** \( \{x\} \rightarrow \{-x\} \rightarrow \{x\} \)

\( \{y\} \rightarrow \{-y\} \rightarrow \{y\} \)
NOTE: Functions with origin symmetry are also even, because they are \( y \)-symmetrical.

When does origin symmetry occur?

<table>
<thead>
<tr>
<th>( x ) symmetrical &amp; ( y ) symmetrical</th>
<th>ALWAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) symmetrical or ( y ) symmetrical</td>
<td>NEVER</td>
</tr>
<tr>
<td>not ( x ) symmetrical, not ( y ) symmetrical</td>
<td>SOMETIMES</td>
</tr>
</tbody>
</table>

example: \( y = x^3 \) \( \rightarrow \) \((-y) = (-x)^3\)
\(-y = (-1)^3(x^3)\)
\(-y = -x^3\)
\(y = x^3 \) \(\leftarrow\) has origin symmetry,
but not \( x \)- or \( y \)-symmetry

**CURVES**

A critical point is a point on a curve where the curve changes tendency.

Critical points are also called "extrema."
A **relative maximum** is the highest point in a part of an interval. It may also be called a "local maximum".

A **relative minimum** is the lowest point in a part of an interval. It may also be called a "local minimum".

A **critical point** can be 1) local maximum, or 2) local minimum.

An **absolute maximum** is the highest point in an interval. It may also be called a "global maximum".

An **absolute minimum** is the lowest point in an interval. It may also be called a "global minimum".

An absolute min/max does not need to be a critical point; it can also be an end point.

**Concavity** is the direction of the curve, up or down:
- $\cup$ is concave up
- $\cap$ is concave down

The term "convexity" is not used.
An inflection point is a point on a curve where the curve changes concavity.

**Example**

LINEAR MODELS

A **mathematical model** is an equation that attempts to represent a natural phenomenon.

A **line** is a model that has 2 **unique characteristics:**

1) **Slope** (steepness or slant)

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

This definition of slope does not work for a vertical line, which has no slope \( \left( \frac{\Delta x}{\Delta y} = \frac{\text{no change}}{\text{infinitely large change}} \right) \).

A horizontal line has a slope \( = 0 \).

2) a point, which may be the y-intercept.

If a line's slope and a point are known, use the point-slope form to find its equation:

\[
(y - y_1) = m(x - x_1)
\]

To find \( b \), which is the y-intercept, set \( x_1 = 0 \).
The x-intercept of a line is called the "zero to the curve." To find it, set $y = 0$.

This is also called finding the root.

**Example**

There are 15 crickets in the yard on day 5 and 20 on day 10. How many are there on day 365? Assume the model is linear.

\[ m = \frac{\Delta x}{\Delta y} = \frac{20 - 15}{10 - 5} = \frac{5}{5} = 1 \]

\[ f(365) = 365 + 10 \]

\[ y - 20 = 1(x - 10) \]

\[ y - 20 = x - 10 \]

\[ y = x + 10 \]

**Power Functions**

A power function is a variable raised to a constant: $f(x) = x^n$

**Example**

$y = x^1$ or $y = x^p$...

\[ \text{power} = 1 \]

\# of x-intercepts = 1

so \# of roots = 1
<table>
<thead>
<tr>
<th>Example</th>
<th>( y = x^2 + \ldots )</th>
<th># of roots = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td>2 real roots</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Note:** Imaginary roots come in pairs, called a "complex conjugate".

<table>
<thead>
<tr>
<th>Example</th>
<th>( y = x^3 + \ldots )</th>
<th># of roots = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Graph" /></td>
<td>3 real roots</td>
<td><img src="image5" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th>( y = x^4 + \ldots )</th>
<th># of roots = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7" alt="Graph" /></td>
<td>4 real roots</td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td><img src="image10" alt="Graph" /></td>
<td>1 double (real) root 2 imaginary or complex roots</td>
<td><img src="image11" alt="Graph" /></td>
</tr>
</tbody>
</table>
A complex number is a combination of real and imaginary numbers.

For odd powers, one arm goes up and one arm goes down. 
\[ \rightarrow \text{therefore it must have at least one real root} \]

For even powers, both arms go up or both arms go down.

**EXponential Functions**

An exponential function is a constant raised to a variable: \( f(x) = a^x \)

example: \( y = 2^x \)
An exponential function always crosses the y-intercept at 1 because $a^0 = 1$.

$$y = a^{-x} \Rightarrow y = \frac{1}{a^x}$$

On the $-x$ axis, this function approaches 0.

**Example**

$$y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

NOTE: A function of form $f(x) = a^{-x}$ never crosses the $x$-axis.

**Example**

$$y = \left(\frac{1}{2}\right)^x$$

$y = 2^{-x}$

$a^{-x}$ is a reflection of $a^x$ over the $y$-axis.

The natural exponential function is $e^x$ where $e = 2.7182818\ldots$ $e^x$ is a function found in nature.

**FACTOR ANALYSIS**

Factor analysis is a method used to determine when an expression is positive or negative.

**Steps:**
1) factor completely
2) evaluate each factor (find the O value; mark the positives/negatives)
3) break the lines into regions
4) mark sign of each region
5) visualize graph
Example

\[
y = \frac{(x+3)(x+2)}{(x-1)}
\]

\[
\begin{array}{cccccccc}
  & & & & & & & \\
-1 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\hline \\
& & & & & & & \\
-1 & - & + & + & + & + & + & + \\
& & & & & & & \\
\end{array}
\]

Example

\[
x^2 - 5x \leq -6
\]

\[
x^2 - 5x + 6 \leq 0
\]

\[
(x-3)(x-2) \leq 0
\]

Why make the right side = 0?

In order to solve for the x's and create factors, since together they must be = 0 or negative. So why? Because 0 is the divider between positive and negative.

\[
\begin{array}{cccccccc}
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
\hline \\
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
\end{array}
\]

\[
2 \leq x \leq 3
\]

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\[
x^2 - 5x = -6
\]

\[
x^2 - 5x + 6 = 0
\]

\[
(x-3)(x-2) = 0
\]

In order to solve for the x's and create factors, since together they must be = 0, so one or both must be 0.

\[x = 3 \text{ or } x = 2\]
Be as efficient as possible when doing factor analysis.

Example:

\[
\frac{x^2+4)(2-x)}{(x-3)^2(x+7)}
\]

A purely positive row has no effect on the sign. Don't include it.

These two rows together will always be positive. Don't include them.

Repetitive signs indicate that there are unnecessary row(s).

Neg \( x<-7, x>2 \)
Pos \(-7<x<2\)
If \( x=2 \), the expression is 0.
If \( x=-7 \), the expression DNE

Example (can't from before)

\( y = |x^2-4| = \begin{cases} (x^2-4) \text{ when } (x^2-4) \geq 0 \\ -(x^2-4) \text{ when } (x^2-4) < 0 \end{cases} \)

Because expression is sometimes positive and sometimes negative, use factor analysis.

\( y = |x^2-4| = \begin{cases} (x^2-4) \text{ when } x \leq -2 \\ -(x^2-4) \text{ when } x \geq 2 \end{cases} \)

Or \([-\infty, -2]\)
\((-2, 2\)
\([2, \infty)\)
TRIGONOMETRY

Trigonometry is the study of triangles.

\[
\begin{align*}
\text{opp} &= \sin \theta = \frac{1}{\csc \theta} \\
\text{hyp} &= \cos \theta = \frac{1}{\sec \theta} \\
\text{adj} &= \tan \theta = \frac{1}{\cot \theta} \\
\text{opp} &= \csc \theta = \frac{1}{\sin \theta} \\
\text{hyp} &= \sec \theta = \frac{1}{\cos \theta} \\
\text{adj} &= \cot \theta = \frac{1}{\tan \theta}
\end{align*}
\]

Note: These are called "trigonometric identities."

What is \( \frac{\sin \theta}{\cos \theta} \)?

\[
\frac{\text{opp}}{\text{hyp}} \div \frac{\text{adj}}{\text{hyp}} = \frac{\text{opp} \cdot \text{hyp}}{\text{hyp} \cdot \text{adj}} = \frac{\text{opp}}{\text{adj}}
\]

So \( \frac{\sin \theta}{\cos \theta} = \tan \theta \)

Consider the Pythagorean theorem:

\[
A^2 + O^2 = H^2
\]

\[
\begin{align*}
\frac{A^2}{A^2} + \frac{O^2}{A^2} &= \frac{H^2}{A^2} \\
\frac{A^2}{H^2} + \frac{O^2}{H^2} &= \frac{H^2}{H^2} \\
\frac{A^2}{O^2} + \frac{O^2}{O^2} &= \frac{H^2}{O^2}
\end{align*}
\]

\[
\begin{align*}
(1) + \left(\frac{O}{A}\right)^2 &= \left(\frac{H}{A}\right)^2 \\
\left(\frac{A}{H}\right)^2 + \left(\frac{O}{H}\right)^2 &= 1 \\
\left(\frac{A}{O}\right)^2 + 1 &= \left(\frac{H}{O}\right)^2
\end{align*}
\]

\[
\begin{align*}
1 + \tan^2 \theta &= \sec^2 \theta \\
\cos^2 \theta + \sin^2 \theta &= 1 \\
\cot^2 \theta + 1 &= \csc^2 \theta
\end{align*}
\]

These are also trigonometric identities.
An **isosceles triangle** has two equal sides and may be a right triangle.

An **equilateral triangle** has three equal sides.

45°, 45°, 90° triangle:

\[ \cos 45° = \frac{1}{\sqrt{2}}, \text{ etc.} \]

30°, 60°, 90° triangle:

\[ a = \frac{\sqrt{3}}{2} \cdot a \]

Circle:

\[ C = 2\pi r \]
\[ A = \pi r^2 \]

\[ \pi = 180° \]
\[ \pi = 200 \text{ grades} \]

A **radian** is the length of the arc.
Unit Circle:

\[ \begin{align*}
\sin \theta & = \frac{y}{r} = y \\
\cos \theta & = \frac{x}{r} = x
\end{align*} \]

so the \( y \) value is the sine of its angle.

so the \( x \) value is the cosine of its angle.

\[ \begin{align*}
\cos 0, \sin 0 & \\
\cos(-1), \sin(-1) & \\
\cos 1, \sin 0 & \\
\cos 0, \sin(-1) &
\end{align*} \]
In the 1st quadrant, all trig functions are positive.

In the 2nd quadrant, only the sine (and its reciprocal cosecant) are positive.

Why? $r$ is not always positive. $r$ is positive in the quadrant where the angle terminates.

See polar coordinates.

Example: $\tan \frac{7\pi}{3}$

$\frac{7}{3} \pi = 2\frac{1}{3} \pi$

so $\tan \frac{7\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example: $\cot \frac{17\pi}{6}$

$\frac{17}{6} \pi = 2\frac{5}{6} \pi$

so $\cot \frac{17\pi}{6} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$
Graph of $\cos x$ using unit circle:
\[y = \cos x\]
Period: $2\pi$

Graph of $\sin x$ using unit circle:
\[y = \sin x\]
Period: $2\pi$

The graphs of sine and cosine vary by a "phase shift" of $\pi/2$.

Graph of $\tan x$ using unit circle:
\[y = \tan x\]
\[\tan \theta = \frac{\sin \theta}{\cos \theta}\]
- At 0, $\tan 0 = 0$
- At $\pi/2$, $\tan \pi/2 = DNE$
- At $\pi$, $\tan \pi = 0$
- At $3\pi/2$, $\tan 3\pi/2 = DNE$
Period: $\pi$

At $\pi/2$ and $3\pi/2$, there are asymptotes.

Graph of $\csc x$ using unit circle:
\[y = \csc x\]
Wherever $\sin x = 0$,
the cosecant has an asymptote.
Period: $2\pi$
Graph of secant using unit circle:

\[ y = \sec x \]

Wherever \( \cos = 0 \), the secant has an asymptote.

Period = \( 2\pi \)

Graph of cotangent using unit circle:

\[ y = \cot x \]

Wherever \( \tan = 0 \), the cotangent has an asymptote.

Period = \( \pi \)

Sine is an odd function

\[ \sin(-x) = -\sin x \]

Cosine is an even function

\[ \cos(-x) = \cos x \]
Cofunctions

Complementary functions (or cofunctions) are equivalent for complementary angles. That is,
\[
\begin{align*}
\sin \theta &= \cos \left( \frac{\pi}{2} - \theta \right) \\
\sec \theta &= \csc \left( \frac{\pi}{2} - \theta \right) \\
\tan \theta &= \cot \left( \frac{\pi}{2} - \theta \right)
\end{align*}
\]

Example:

\[
\begin{array}{c}
\text{30° and 60° are complements} \\
\text{so } \sin 30° = \cos 60° \\
\frac{1}{2} = \frac{1}{2}
\end{array}
\]

The law of sines and the law of cosines can be used to solve triangles that are not right (and right triangles too).

df. Law of sines: \[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

df. Law of cosines: \[
C^2 = A^2 + B^2 - 2AB \cos C
\]

If the triangle is right and \( C \) is set at the 90° angle, then \(-2AB \cos C = 0\), leading us to the Pythagorean theorem.

If you know an angle and its side opposite, use the law of sines.
If you know an angle and its side adjacent, use the law of cosines.
INVERSE FUNCTIONS

An inverse function is the changing of the independent and dependent variables (with each other) of the original function which graphically reflects the function about the line $y = x$.

The input becomes the output, and the output becomes the new input.

If the original equation was a function, then the inverse may also be a function (the "inverse function").

Example: $f(x) = 5x + 2 \Rightarrow x = 5y + 2$

$5y = x - 2$

$y = \frac{x - 2}{5} = f'(x)$

An inverse is NOT a reciprocal. $f'(x) \neq \frac{1}{f(x)}$

Geometrically, the inverse function is the reflection of the function over $x = y$.

Example

Not all inverses of functions are functions.
Every trig function has an inverse function. See pp. 72 in the book for their graphs.

How are inverses handled if they are not functions? Truncate them to a smaller interval that is a function.

Each inverse trig function has a restriction. For example, $\cos^{-1}x \quad 0 < y < \pi$
- $-1 < x < 1$

The reciprocals $\csc^{-1}$ and $\sec^{-1}$ have the same range (held between asymptotes) as their counterparts.
But \( \cot^{-1} \) is different. Its range is \( 0 \text{ to } \pi \),
or \( -\frac{\pi}{2} \text{ to } \frac{\pi}{2} \),
not including 0
(which is an asymptote).

\[ (0, \pi) \]
\[ \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \]
not incl. 0

Example
\[ \theta = \sin^{-1} \frac{1}{\sqrt{2}} \]
(1) redefine it: \( \sin \theta = \frac{1}{\sqrt{2}} \)
(2) graph it:

Why is \( \theta \) in the first quadrant?
• \( \sin \theta \) is positive, so it must be in the 1st or 4th quadrants
• \( \sin^{-1} \theta \) is positive, so it must be in the 1st or 2nd quadrants
∴ it's in the 1st

\[ \sin (\sin^{-1} x) = x \]

\[ \text{NOTE that the inverse sine is an angle, not a value.} \]
example \hspace{1cm} \tan(\sin^{-1}(-\frac{1}{\sqrt{2}}))

from the previous example, we know \(\sin^{-1}(\frac{1}{\sqrt{2}}) = -\frac{\pi}{4}\)

so \(\tan(-\frac{\pi}{4}) = -1\)

---

**LOGARITHMS**

**df.** A logarithm is an exponent.

example \hspace{1cm} 1000 = 10^x \hspace{1cm} so \(x = 3\)

this is equivalent to \(\log_{10} 1000 = 3\)

example \hspace{1cm} 586 = 10^x \hspace{1cm} what is \(x\)? \(\log_{10} 586 = x\)

\(\log_{10}\) is called the "common log" (because our numbering system is base 10).

The other log found on a calculator is \(\log_{e}\), called the "natural log". It is also written as \(\ln\).

example \hspace{1cm} (586 \cdot 3197) \approx (10^{2.7479} \cdot 10^{-0.4752}) \approx 10^{2.2727}

\(\approx 187.284\)

Logs are always positive because there is no exponent of a positive base that will generate a negative number.
Laws of Logs —

\[ A = B^c \] (exponential form) is the same as
\[ \log_b A = c \] (logarithmic form)

\[ \log_c (A \cdot B) = \log_c A + \log_c B \]

\[ \frac{\log_c A}{\log_c B} = \log_c A - \log_c B \]

\[ \log_c (A^b) = b \log_c A \]

\[ \log_a A = 1 \quad (a^x = a \Rightarrow x = 1) \]

\[ \log_a 1 = 0 \quad (a^x = 1 \Rightarrow x = 0) \]

\[ \log_a 0 = \text{DNE} \]

\[ \log_a A^b = b \log_a A \]

\[ \log_a A = \ln A \]

\[ \ln e = 1 \quad (e^x = e \Rightarrow x = 1) \]

\[ \ln 1 = 0 \quad (e^x = 1 \Rightarrow x \neq 0) \]

\[ \ln e^A = A \]

\[ e^{\ln x} = x \]
What if a problem uses a base other than 10 or $e$?

**Base conversion formula:**

$$\log_b A = \frac{\log_c A}{\log_c B} = \frac{\log_{10} A}{\log_{10} B} = \frac{\ln A}{\ln B}$$

**Example**

$$\log_2 5 = \frac{\log 5}{\log 2} = \frac{\ln 5}{\ln 2}$$

---

**TRANSFORMATIONS**

**Translations**

- **y direction**
  - $y + k = f(x)$ → down by $k$
  - $y - k = f(x)$ → up by $k$

- **x direction**
  - $y = f(x + h)$ ← left by $h$
  - $y = f(x - h)$ → right by $h$

Think of "$k" and "$h" as the motion of the coordinate axis.

**Deformations**

- **y direction**
  - $cy = f(x)$ → stretch by $c$
  - $\frac{1}{c} y = f(x)$ → shrink by $c$

- **x direction**
  - $y = f(cx)$ → stretch by $c$
  - $y = f(\frac{x}{c})$ ← shrink by $c$

**Reflection**

- $-y = f(x)$ → reflect over the x-axis
- $y = f(-x)$ → reflect over the y-axis
\[ y = -3(2x - 4)^3 - 3 \]

The parent is \( y = x^3 \)

\[ y = -3(2x - 4)^3 - 3 \]
\[ y + 3 = -3(2x - 4)^3 \]
\[ -\frac{1}{3}(y + 3) = (2x - 4)^3 \]
\[ -\frac{1}{3}(y + 3) = (2(x - 2))^3 \]

To graph:
1. \( y = 2x^3 \) compress by 2 in \( x \)
2. \( y = (2(x-2))^3 \) to right by 2
3. \( \frac{1}{3}y = (2(x-2))^3 \) stretch by 3 in \( y \)
4. \( -\frac{1}{3}y = (2(x-2))^3 \) reflect over \( x \)
5. \( -\frac{1}{3}(y + 3) = (2(x-2))^3 \) down by 3

or just use tracer points:

6. \( y = x^3 \)  \((-1, -1)\)  \((0, 0)\)  \((1, 1)\)
7. \( y = 2x^3 \)  \((-\frac{1}{2}, -1)\)  \((0, 0)\)  \((\frac{1}{2}, 1)\)
8. \( y = (2(x-2))^3 \)  \((\frac{3}{2}, -1)\)  \((2, 0)\)  \((\frac{5}{2}, 1)\)
9. \( \frac{1}{3}y = (2(x-2))^3 \)  \((\frac{3}{2}, -3)\)  \((2, 0)\)  \((\frac{5}{2}, 3)\)
10. \( -\frac{1}{3}y = (2(x-2))^3 \)  \((\frac{3}{2}, 3)\)  \((2, 0)\)  \((\frac{5}{2}, -3)\)
11. \( -\frac{1}{3}(y + 3) = (2(x-2))^3 \)  \((\frac{3}{2}, 0)\)  \((2, -3)\)  \((\frac{5}{2}, -6)\)