Foundation for the statistical treatment of Matter

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Motivation

Quantum Theory has proven a useful predictive tool for working with matter. However an issue that most often remains unattended to is the core meaning behind the theory. Instead of having a foundational theory to guide the development, various philosophical points of view are put forth such as wave-particle duality or the many worlds interpretation. In this piece the notion of matter's wave-particle duality will be rejected in favor of a more statistical viewpoint. This piece will begin with a brief introduction of probabilities before leading into the behavior of matter in general. The results will be familiar to anyone who studies Quantum Theory.

Basic statistics with probabilities

First consider a series of N measurements with A and B representing a potential discrete outcomes. The outcomes resulting in A can be expressed in a the proportion of all measurements. As the size of the measurement sample goes to ∞ then this proportion would approach a number that can be called the *probability* of outcome A.

$$P(A)=\lim_{N\to\infty}\frac{N_A}{N} \tag{1}$$

This same line of thought can be extended to cases where one wants both conditions A and B to be satisfied.

$$P(A\text{ and }B)=\lim_{N\to\infty}\frac{N_{AB}}{N} \tag{2}$$

Likewise one can speak of the probability that either A or B will be realized. However one must use caution in using the definition set up in (1) to not over-count the cases where both A and B are realized.

$$P(A\text{ or }B)=\lim_{N\to\infty}\frac{N_A+N_B-N_{AB}}{N}$$

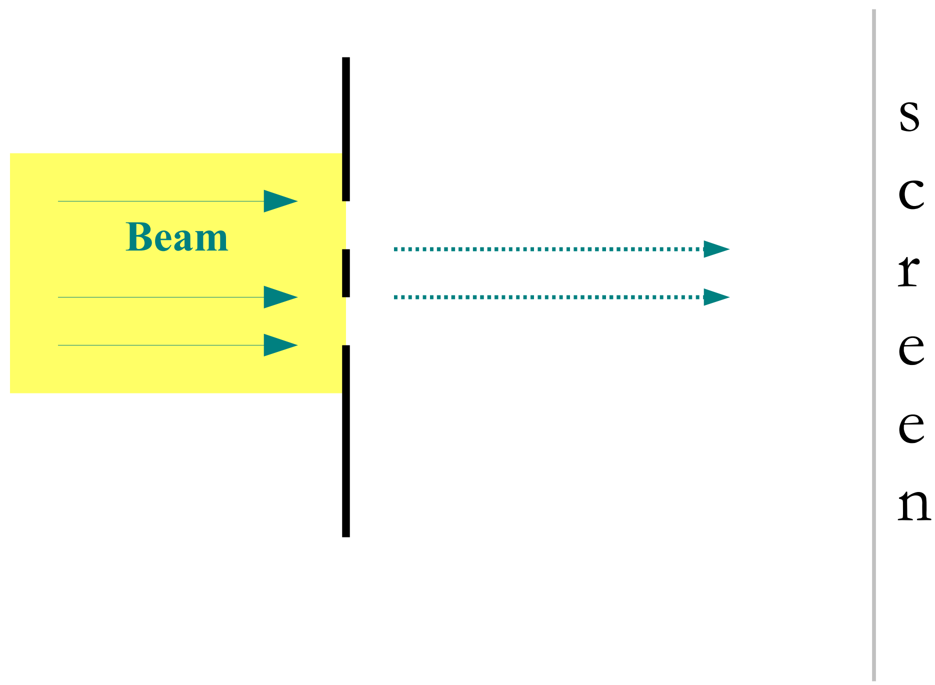
$$P(A\text{ or }B)=P(A)+P(B)-P(A\text{ and }B) \tag{3}$$

The two-slit experiment

Consider a beam consisting of bits of some form of matter passing through a barrier with two narrow slits placed close by each other. The beam that passes through the slits is then projected onto a screen as shown in the figure on the following page.

What would one expect the observed pattern to be with regards to where the bits of matter will end up on the screen? We'll draw upon (3) to answer this question because we presume that the matter will hit a point on the screen after having passed through one slit or another but not both. Moreover the positions that the matter can be found on the screen will lie on a continuum of possibilities rather than a discrete set. Therefore probability densities ρ need to be employed rather than probabilities themselves.

$$\rho(x) = \rho_1(x) + \rho_2(x) \tag{4}$$



Equation (4) suggest a bimodal distribution on the screen, one distribution for each slit. The trouble however is that it does not always work. Sometimes the distribution has multiple regions of high probability, and this is something that (4) cannot deliver.

What went wrong

Without intending to, an assumption was made in obtaining (4) for application to the two-slit experiment, and this is that the physics working on our matter is local. In other words the measurement in one part of the experiment does not simultaneously alter the rest of the experiment. Indeed when we stated earlier that the distribution of measurements would be bimodal, we inherently assumed a sum of distributions that would be the same independent of whether any measurement at the slits had been made. In this vein we could take the probabilities that the matter would reach a portion of the screen assumed that we knew which slit it passed through and use these even when such a measurement had not in fact taken place. The task is now to reconstruct the statistics without making this assumption. At the same time making sure that our modified statistical treatment can account for the outcomes of the above experiment.

Working towards a new statistical model

In developing a new statistic a few criteria must be insisted upon.

- We still want to be able to use the probabilities via the two slits as before.

- The equation cannot produce negative probabilities.

- Equation (4) can still be a valid equation in a special case, so we must allow for it to be a possible outcome.

- If the matter cannot reach a point via one slit then the final probability must equal that of the other slit. For example if $\rho_1(x)=0$ then $\rho(x)=\rho_2(x)$.

The first criteria can be satisfied by applying a correction term to (4).

$$\rho(x)=\rho_1(x)+\rho_2(x)+\Delta\rho(x) \tag{5}$$

To further aide in the task before us, consider three cases.

Equation (4) is valid

In this case the outcome must yield an earlier conclusion with $\Delta\rho = 0$.

$$\rho_0(x)=\rho_1(x)+\rho_2(x) \tag{6a}$$

Minimum probability

What is the minimal probability density possible for a given $\rho_1(x)$ and $\rho_2(x)$, other than zero, that matches our criteria? The simplest candidate is

$$\rho_{min}(x)=\left(\sqrt{\rho_1(x)}-\sqrt{\rho_2(x)}\right)^2$$

$$\rho_{min}(x)=\rho_1(x)+\rho_2(x)-2\sqrt{\rho_1(x)\cdot\rho_2(x)} \tag{6b}$$

The subtraction serves to minimize the probability while the square eliminates any negative results. Moreover the last term in (6b) can be said to take on the role of the correction term in (5).

Maximum probability

So now what could be the maximum probability density? There is no definite upper limit as to what a probability density can be, however one can safely deduce that in this case the correction term in (5) would have to be positive and so.

$$\rho_{max}(x)\geq\rho_1(x)+\rho_2(x) \tag{6c}$$

It must be pointed out that while (6c) is open ended due to the inequality, on average (5) must still be consistent with (4).

Putting equations (5), (6a), (6b) and (6c) together suggests a methodology for combining probabilities.

$$\rho(x)=\rho_1(x)+\rho_2(x)+2\sqrt{\rho_1(x)\cdot\rho_2(x)}\cdot\cos\Theta \qquad (7)$$

A new statistical view of matter

Equation (7) ties together all the previous pieces, and yet is still unsatisfactory. The main reason is that it introduces a Θ that is an unspecified quantity.

What is needed is an improved relation. To do so a lesson can be taken from (6b) that allows for generalization. Define **Ψ** to be a complex number such that

$$\rho(x)=|\Psi(x)|^2$$

Let this new quantity be called the *probability amplitude*. Then let (7) be recreated by simply adding these amplitudes.

$$\Psi=\Psi_1+\Psi_2 \qquad (8)$$

$$|\Psi|^2=|\Psi_1+\Psi_2|^2$$

$$\rho(x)=|\Psi_1|^2+|\Psi_2|^2+2|\Psi_1|\cdot|\Psi_2|\cos(\phi_1-\phi_2)$$

$$\rho(x)=\rho_1(x)+\rho_2(x)+2\sqrt{\rho_1(x)\cdot\rho_2(x)}\cos(\phi_1-\phi_2) \qquad (9)$$

Here ϕ_1 and ϕ_2 are the complex phases. It can be seen by inspection that (9) is consistent with (7). Moreover the unknown Θ is now given significance as the difference in the complex phase of the respective probability amplitudes.

Illustration: The toss of a die

Consider how the above can be applied to the real-world example of computing dice probability. In particular, the probability that a die toss will produce either a 1 or a 6 roll. For simplicity let us assume that all amplitudes are either positive or negative but not complex.

Consistent with our earlier definition our probability amplitudes will be taken to be

$$\Psi_1=\frac{\pm 1}{\sqrt{6}} \qquad (10a)$$

$$\Psi_6=\frac{\pm 1}{\sqrt{6}} \qquad (10b)$$

We have two possible outcomes, one where the signs are the same and one where they are different.

$$P_{same}=\left|\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{6}}\right|^2=\frac{2}{3} \qquad (11a)$$

$$P_{diff} = \left| \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right|^2 = 0 \tag{11b}$$

While these answers may not look right they in fact are. Consider that for a large number of rolls (a requirement if we are to speak of probabilities) both scenarios are equally likely. So the empirical probability obtained will be the average of the two.

$$P = \frac{1}{2}(P_{same} + P_{diff}) = \frac{1}{3} \tag{12}$$

This last result is indeed the expected outcome.

Conclusion

The behavior of matter is different then what on might expect based on common experience. Specifically the statistical nature of matter behaves in such a manner that probabilities can not, in general, be added together as previously supposed. Rather one can work instead with probability amplitudes Ψ defined such that:

$$|\Psi|^2 = P$$

We add probability amplitudes largely the same as the probabilities earlier, except these amplitudes need not be positive numbers at all.

A more detailed analysis of the consequences is beyond the scope of this piece.

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