

A **Rational Expression** is a $\frac{\text{Polynomial}}{\text{Polynomial}}$

For example:

$$\frac{x-2}{4}, \frac{4}{x-2}, \frac{x}{x^2-1}, \text{ and } \frac{x^2+1}{x^2+2x-3}$$

A **Complex Fraction** is a $\frac{\text{Rational Expression}}{\text{Rational Expression}}$ or it is a $\frac{\text{fraction}}{\text{fraction}}$

In Problems 17-34, perform the indicated operation and simplify the result. Leave your answer in factored form.

24. $\frac{x^2+x-6}{x^2+4x-5} \cdot \frac{x^2-25}{x^2+2x-15}$

$$\frac{(x+3)(x-2)}{(x+5)(x-1)} \cdot \frac{(x-5)(x+5)}{(x+5)(x-3)}$$

$$\frac{(x+3)(x-2)(x-5)}{(x+5)(x-1)(x-3)}$$

34.

$$\frac{9x^2+3x-2}{12x^2+5x-2} \div \frac{9x^2-6x+1}{8x^2-10x-3}$$

$$\frac{(3x-1)(3x+2)}{(3x+2)(4x-1)} \cdot \frac{(4x+1)(2x-3)}{(3x-1)(3x-1)}$$

$$= \frac{(4x+1)(2x-3)}{(4x-1)(3x-1)}$$

In Problems 35-52, perform the indicated operations and simplify the result. Leave your answer in factored form.

$$\begin{aligned}
 42. \quad \frac{5x-4}{3x+4} - \frac{(x+1)}{3x+4} &= \frac{5x-4-(x+1)}{3x+4} && \text{LCD: } 3x+4 \\
 &= \frac{5x-4-x-1}{3x+4} \\
 &= \frac{4x-5}{3x+4}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{6}{x-1} - \frac{x}{1-x} \\
 \frac{6}{x-1} - \frac{-x}{-1(1-x)} \\
 \frac{6}{x-1} - \frac{-x}{x-1} \\
 \frac{6 - -x}{x-1} = \frac{6+x}{x-1}
 \end{aligned}$$

What is a Least Common Multiple (LCM)? The Least Common Multiple (LCM) is the smallest algebraic expression that two or more algebraic expressions will divide into evenly.

In Problems 53-60, Find the LCM of the given polynomials

56. $3x^2 - 27$, $2x^2 - x - 15$

$$\begin{array}{l}
 3(x^2 - 9), (2x + 5)(x - 3) \\
 3(x + 3)(x - 3), (2x + 5)(x - 3) \\
 \text{LCD: } \underline{3(x - 3)(x + 3)(2x + 5)}
 \end{array}$$

In Problems 61-72. Perform, the indicated operations and simplify the result. Leave your answer in factored form.

68. $\frac{2x - 3}{x^2 + 8x + 7} - \frac{x - 2}{(x + 1)^2}$

$$\begin{array}{l}
 \text{LCD: } x^2 + 8x + 7; (x + 1)^2 \\
 (x + 1)(x + 7); (x + 1)(x + 1) \\
 \underline{(x + 1)(x + 7)(x + 1)}
 \end{array}$$

$$\frac{(2x - 3)(x + 1)}{(x + 1)(x + 7)(x + 1)} - \frac{(x - 2)(x + 7)}{(x + 1)(x + 1)(x + 7)}$$

$$\frac{(2x - 3)(x + 1) - (x - 2)(x + 7)}{\text{LCD}}$$

$$\frac{2x^2 - 3x + 2x - 3 - (x^2 - 2x + 7x - 14)}{\text{LCD}}$$

$$\frac{2x^2 - x - 3 - x^2 + 2x - 7x + 14}{\text{LCD}}$$

$$\frac{x^2 - 6x + 11}{(x + 1)(x + 1)(x + 7)}$$

In Problems 73-82. Perform the indicated operations and simplify the result. Leave your answer in factored form.

74.

$$\frac{4 + \frac{1}{x^2}}{3 - \frac{1}{x^2}}$$

$$\frac{\frac{4x^2 + 1}{x^2}}{\frac{3x^2 - 1}{x^2}}$$

Invert
&
multiply

$$\frac{4x^2 + 1}{x^2} \cdot \frac{x^2}{3x^2 - 1} =$$

$$\frac{4x^2 + 1}{3x^2 - 1}$$

Numerator: LCD x^2

$$\frac{x^2 4 + \frac{1}{x^2}}{x^2} = \frac{4x^2 + 1}{x^2}$$

Denominator LCD x^2

$$\frac{x^2 3 - \frac{1}{x^2}}{x^2} = \frac{3x^2 - 1}{x^2}$$

76.

$$\frac{1 - \frac{x}{x+1}}{2 - \frac{x-1}{x}}$$

$$\frac{\frac{1}{x+1}}{\frac{x+1}{x}}$$

$$\frac{1}{x+1} \cdot \frac{x}{x+1} = \frac{x}{(x+1)^2}$$

Numerator: LCD: $x+1$

$$\frac{(x+1)1 - \frac{x}{x+1}}{(x+1)} = \frac{x+1 - x}{x+1} = \frac{1}{x+1}$$

Denominator: LCD: x

$$\frac{x^2 - \frac{x-1}{x}}{x} = \frac{2x - (x-1)}{x}$$

$$= \frac{2x - x + 1}{x}$$

$$= \frac{x+1}{x}$$

Extras from A8 --- Solving Rational Equations ← multiply Both Sides by LCD

38. $\frac{-2}{x+4} = \frac{-3}{x+1}$

LCD: ~~x+4~~, x+1

$$\cancel{(x+4)}(x+1) \left[\frac{-2}{\cancel{x+4}} \right] = \cancel{(x+4)}(x+1) \left[\frac{-3}{\cancel{x+1}} \right] \frac{(x+4)(x+1)}{(x+4)(x+1)}$$

$$-2(x+1) = -3(x+4)$$

$$-2x - 2 = -3x - 12$$

$$x - 2 = -12$$

$$x = -10$$

$x \neq -4$
 $x \neq -1$
can't divide by 0.

40. $\frac{1}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)}$

LCD: ~~(2x+3)~~(x-1)

$$\cancel{(2x+3)}(x-1) \left[\frac{1}{\cancel{2x+3}} \right] + \cancel{(2x+3)}(x-1) \left[\frac{1}{\cancel{x-1}} \right] = \cancel{(2x+3)}(x-1) \left[\frac{1}{\cancel{(2x+3)(x-1)}} \right]$$

$$1(x-1) + 1(2x+3) = 1$$

$$x-1 + 2x+3 = 1$$

$$3x+2 = 1$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$x \neq -\frac{3}{2}$
 $x \neq 1$