Circles:

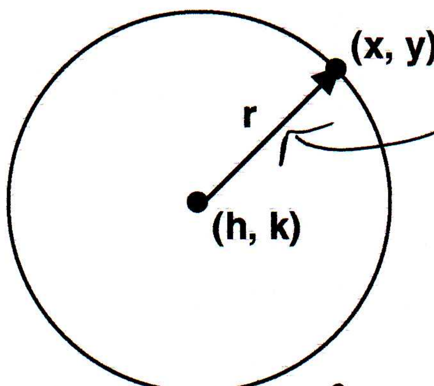
A ***Circle*** is a set of points in the xy-plane that are a fixed distance r from a fixed point **(h, k)**. The fixed distance r is called the ***radius*** and the fixed point **(h, k)** is called the ***center of the circle***.

Equation of a Circle (Standard Form)

$$(x-h)^2+(y-k)^2=r^2$$

is an equation of the circle with radius r and center at (h, k).

Development of the Above Formula



distance
a constant.

r is the distance from center (h, k) to any point on the circle. (x, y)

$$r=\sqrt{(x-h)^2+(y-k)^2}$$

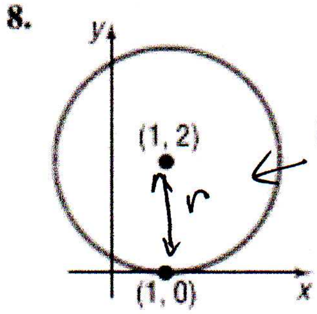
$$r^2=\left(\sqrt{(x-h)^2+(y-k)^2}\right)^2$$

$$r^2=(x-h)^2+(y-k)^2$$

which is the standard form for the circle.

In Problems 5-8, Find the center and radius of each circle. Write the standard form of the equation.

8.



← line vertical so $r = 2$

$h \quad k$

C: $(1, 2)$ $\qquad r = 2$

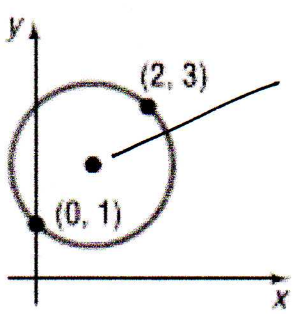
$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-2)^2=2^2$

or

$\boxed{(x-1)^2+(y-2)^2=4}$

10.



need center. it is the mid point between $(0,1)$ + $(2,3)$

$(h, k) = \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$

C: $(1, 2)$ Now we need the radius. It is the distance between $(0,1)$ + $(1,2)$

$r = \sqrt{(1-0)^2+(2-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

C: $(1, 2)$ $\qquad r = \sqrt{2}$

$(x-1)^2+(x-2)^2=(\sqrt{2})^2$

$\boxed{(x-1)^2+(x-2)^2=2}$

In Problems 11-20, write the standard from of the equation and the general form of the equation of each circle of radius r and center (h, k). Graph each circle.

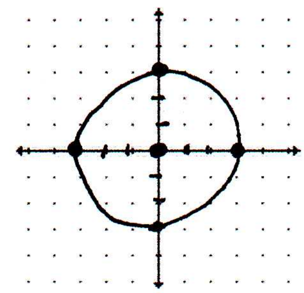
h

12. $r = 3$; $(h, k) = (0, 0)$

$$(x-0)^2+(y-0)^2=3^2$$

$$\boxed{x^2+y^2=9}$$

$$\boxed{x^2+y^2-9=0}$$



16. $r = 4$; $(h, k) = (2.–3)$

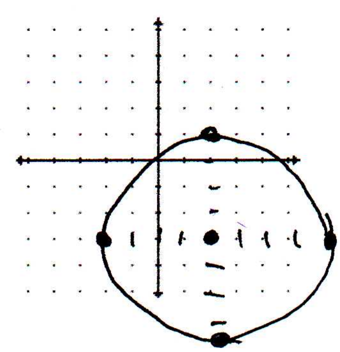
$$(x-2)^2+(y--3)^2=4^2$$

$$\boxed{(x-2)^2+(y+3)^2=16}$$

$$x^2-4x+4+y^2+6y+9=16$$

$$x^2+y^2+4x-6y+13=16$$

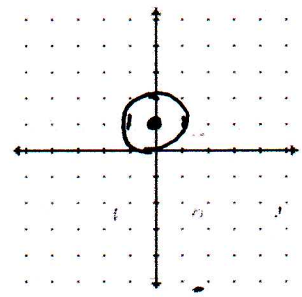
$$\boxed{x^2+y^2+4x-6y-3=0}$$



In Problems 21-34, find the center (h, k) and radius r of each circle. Graph each circle.

22. $x^2 + (y-1)^2 = 1$

$0 \quad 1$



Center: $(0, 1)$

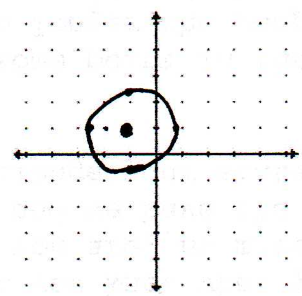
Radius: $\sqrt{1} = 1$

24. $3(x+1)^2 + 3(y-1)^2 = 6$

$\frac{3(x+1)^2}{3} + \frac{3(y-1)^2}{3} = \frac{6}{3}$

$(x+1)^2 + (y-1)^2 = 2$

$-1 \quad 1$



Center: $(-1, 1)$

Radius: $\sqrt{2} \approx 1.4$

How to Complete the Square: **Creating a perfect trinomial square of the form: $(x-p)^2$**

We want to turn x^2+bx+_____ into a perfect trinomial square $(x+p)^2$ by adding the correct quantity.

Formula: $\left(\frac{b}{2}\right)^2$

1 $x^2+2x + 1 = (x+1)^2$

$\left(\frac{2}{2}\right)^2 = 1$

2. $x^2-10x + 25 = (x-5)^2$

$\left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$

3. $x^2+9x + \frac{81}{4} = \left(x+\frac{9}{2}\right)^2$

$\left(\frac{9}{2}\right)^2 = \frac{81}{4}$

The Process:

Taking $x^2+y^2+bx+cy+d=0$ (General Form) and turning it into $(x-h)^2+(y-k)^2=r^2$ (Standard form)

0. Make sure the coefficients of x^2 and y^2 are 1, if not divide it out.

1. Move the constant to the right hand side

2. Group the x's together and the y's together

3. Complete the square on the x's and add that number to both sides

4. Complete the square on the y's and add that number to both sides

5. Rewrite the equation as $(x-h)^2+(y-k)^2=r^2$