Radical Notation:---- Vocabulary

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Radical $\rightarrow \sqrt[n]{a}$ — Radicand

Properties of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ $\qquad\qquad$ $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

In Problems 7-42, evaluate each perfect root.

14. $\sqrt[4]{48x^5}$

$\sqrt[4]{16x^4 \cdot 3x}$

$\sqrt[4]{16x^4}\ \sqrt[4]{3x}$

$2x\ \sqrt[4]{3x}$

18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}$ reduce first

$\sqrt[3]{\dfrac{1}{27x^3}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{27x^3}} = \dfrac{1}{3x}$

22. $\sqrt{5x}\ \sqrt{20x^3}$ bring Together

$\sqrt{5x \cdot 20x^3}$

$\sqrt{100x^4}$

$10x^2$

30. $2\sqrt{12} - 3\sqrt{27}$ simplify first

$2\sqrt{4 \cdot 3} - 3\sqrt{9 \cdot 3}$

$2\sqrt{4}\sqrt{3} - 3\sqrt{9}\sqrt{3}$

$2 \cdot 2\sqrt{3} - 3 \cdot 3\sqrt{3}$

$4\sqrt{3} - 9\sqrt{3} = -5\sqrt{3}$

Rationalizing Denominators Containing Two Terms How can we rationalize a denominator if the denominator contains two terms with one or more square roots? **Multiply the numerator and the denominator by the conjugate of the denominator.**

Rationalize the denominator: $\dfrac{7}{5+\sqrt{3}}$.

SOLUTION The conjugate of the denominator is $5-\sqrt{3}$. If we multiply the numerator and the denominator by $5-\sqrt{3}$, the denominator will not contain a radical. Therefore, we multiply by 1, choosing $\dfrac{5-\sqrt{3}}{5-\sqrt{3}}$ for 1.

$$\frac{7}{5+\sqrt{3}} = \frac{7}{5+\sqrt{3}}\cdot\frac{5-\sqrt{3}}{5-\sqrt{3}} \qquad \text{Multiply by 1.}$$

$$= \frac{7(5-\sqrt{3})}{5^2-(\sqrt{3})^2} \qquad (A+B)(A-B) = A^2 - B^2$$

$$= \frac{7(5-\sqrt{3})}{25-3} \qquad (\sqrt{3})^2 = \sqrt{3}\cdot\sqrt{3} = \sqrt{9} = 3$$

$$= \frac{7(5-\sqrt{3})}{22} \quad \text{or} \quad \frac{35-7\sqrt{3}}{22}$$

In Problems 43-54. Rationalize the denominator.

48. $\dfrac{\sqrt{2}}{(\sqrt{7}+2)}\dfrac{(\sqrt{7}-2)}{(\sqrt{7}-2)}$ ← Multiply by Conjugate $\sqrt{7}-2$

$$\frac{\sqrt{2}(\sqrt{7}-2)}{\sqrt{7}\sqrt{7}\cancel{+2\sqrt{7}}\cancel{-2\sqrt{7}}-4}$$

$$\frac{\sqrt{2}\sqrt{7}-2\sqrt{2}}{\sqrt{49}-4}$$

$$\frac{\sqrt{14}-2\sqrt{2}}{7-4} = \frac{\sqrt{14}-2\sqrt{2}}{3}$$

Factoring "BLOBS" – and fraction and negative exponents -- an explanation of the process.

Concept: $2x^2+10x^3 = 2x^2(x^{2-2}+5x^{3-2})$

$= 2x^2(x^0+5x^1)$

$= 2x^2(1+5x) \Leftarrow$

$2x^{\frac{1}{2}}+10x^{\frac{3}{2}} = 2x^{\frac{1}{2}}(x^{\frac{1}{2}-\frac{1}{2}}+5x^{\frac{3}{2}-\frac{1}{2}})$

$= 2x^2(x^0+5x^{3/2})$

$= 2x^2(1+5x) \Leftarrow$

$2x^{1/2}+10x^{-3/2} = 2x^{-3/2}(x^{1/2--3/2}+5x^{-3/2--3/2})$

$= 2x^{-3/2}(x^{\frac{4}{2}}+5x^0)$

$= 2x^{-3/2}(x^2+5) \Leftarrow$

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factoring using Substitution:

$2(2x-1)^2+10(2x-1)^3$ let $u=(2x-1)$

$2\ u^2 + 10\ u^3$

$2u^2(1+5u)$

re substitute

$2(2x-1)^2(1+5(2x-1))$

$2(2x-1)^2(1+10x-5)$

$2(2x-1)^2(10x-4)$

$2(2x-1)^2 \cdot 2(5x-2)$

$4(2x-1)^2(5x-2) \Leftarrow$

Practice:

REMINDER: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

1. $4(x+1)^2 - 12(x+1)$ $\quad u=(x+1)$

$4u^2 - 12u$

$4u(u-3)$

$4(x+1)((x+1)-3)$

$4(x+1)(x+1-3)$

$4(x+1)(x-2)$

2. $(4x+1)^3 - 8$ $\quad u=(4x+1)$

$u^3 - 8$

$a = u \quad 2 = b$

$(a-b)(a^2+ab+b^2)$

$(u-2)((u)^2+(u)(2)+(2)^2)$

$(u-2)(u^2+2u+4)$

$((4x+1)-2)((4x+1)^2+2(4x+1)+4)$

$(4x-3)(16x^2+8x+1+8x+2+4)$

$\underline{(4x-3)(16x^2+16x+7)} \Leftarrow$

From Textbook:

92. $6x^{1/2}(2x+3)+x^{3/2}\cdot 8$, $\quad x \geq 0$

$2x^{1/2}\left[3x^{1/2-1/2}(2x+3) + x^{3/2-1/2}\cdot 4\right]$

$2x^{1/2}\left[3(2x+3)+4x\right]$

$2x^{1/2}\left[6x+9+4x\right]$

$2x^{1/2}\left[10x+9\right]$

98. $8x^{1/3} - 4x^{-2/3}$, $\quad x \neq 0$

$4x^{-2/3}\left[2x^{1/3 - -2/3} - x^{-2/3 - -2/3}\right]$

$4x^{-2/3}\left[2x-1\right] \Leftarrow$

76. $\dfrac{1+x}{2x^{1/2}} + x^{1/2}$, $x > 0$

$$\frac{1+x}{2x^{1/2}} + \frac{x^{\frac{1}{2}}}{1} \cdot \frac{2x^{1/2}}{2x^{1/2}}$$

$$\frac{1+x+2x}{2x^{1/2}} = \frac{3x+1}{2x^{1/2}} \Leftarrow$$

94. $2x(3x+4)^{4/3} + x^2 \bullet 4(3x+4)^{1/3}$

94. $2x(3x+4)^{4/3} + x^2 \bullet 4(3x+4)^{1/3}$

$u = (3x+4)$

$$2x\,u^{4/3} + 4x^2\,u^{1/3}$$

$$2xu^{1/3}\left[u^{4/3-1/3} + 2x\right]$$

$$2xu^{1/3}\left[u + 2x\right]$$

$$2xu^{1/3}\left[(3x+4) + 2x\right]$$

$$2x(3x+4)(5x+4) \Leftarrow$$

90. $(x^2+4)^{4/3} + x \cdot \frac{4}{3}(x^2+4)^{1/3} \cdot 2x$ $\qquad u = x^2+4$

$u^{4/3} + x \cdot \frac{4}{3} u^{1/3} \cdot 2x$

$u^{4/3} + \frac{8}{3}x^2 u^{1/3}$

$u^{1/3}\left[u^{4/3-\frac{1}{3}} + \frac{8}{3}x^2 u^{1/3-1/3}\right]$

$u^{1/3}\left[\frac{3u}{3} + \frac{8}{3}x^2\right] \Leftarrow$ Put over common denominator

$u^{1/3}\left[\frac{3u+8x^2}{3}\right]$

$(x^2+4)^{\frac{1}{3}}\left[\frac{3(x^2+4)+8x^2}{3}\right] = (x^2+4)^{\frac{1}{3}}\left[\frac{3x^2+12+8x^2}{3}\right]$

$= (x^2+4)^{\frac{1}{3}}\left[\frac{11x^2+12}{3}\right] \Leftarrow$

96. $6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2}$ $\qquad x \ge \frac{3}{4}$ $\qquad u = (6x+1)$ $\quad V = (4x-3)$

$6u^{1/3}V^{3/2} + 6u^{4/3}V^{1/2}$

$6u^{1/3}V^{1/2}\left[V^{3/2-1/2} + u^{4/3-\frac{1}{3}}\right]$

$6u^{1/3}V^{1/2}(V+u)$

$6(6x+1)^{1/3}(4x-3)^{1/2}(4x-3+6x+1)$

$6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2)$

$6(6x+1)^{1/3}(4x-3)^{1/2} \cdot 2(5x-1)$

$12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1) \Leftarrow$