Synthetic Division

Dividing Polynomials Using Synthetic Division We can use **synthetic division** to divide polynomials if the divisor is of the form $x - c$. This method provides a quotient more quickly than long division. Let's compare the two methods showing $x^3 + 4x^2 - 5x + 5$ divided by $x - 3$.

Long Division

$$\begin{array}{r} x^2 + 7x + 16 \\ x - 3\,\overline{)\,x^3 + 4x^2 - 5x + 5} \\ \ominus x^3 \oplus\!- 3x^2 \qquad\qquad \\ \hline 7x^2 - 5x \qquad \\ \ominus 7x^2 \oplus\!- 21x \qquad \\ \hline 16x + 5 \\ \ominus 16x \oplus\!- 48 \\ \hline 53 \end{array}$$

Quotient

Dividend

Divisor $x - c$; $c = 3$

Remainder

Synthetic Division

3	1	4	−5	5
		3	21	48
	1	7	16	53

Notice the relationship between the polynomials in the long division process and the numbers that appear in synthetic division.

These are the coefficients of the dividend $x^3 + 4x^2 - 5x + 5$.

The divisor is $x - 3$. This is 3, or c, in $x - c$.

3	1	4	−5	5
		3	21	48
	1	7	16	53

These are the coefficients of the quotient $x^2 + 7x + 16$.

This is the remainder.

Now let's look at the steps involved in synthetic division.

SYNTHETIC DIVISION To divide a polynomial by $x - c$:

Example

1. Arrange polynomials in descending powers, with a 0 coefficient for any missing term.

$$x - 3\overline{)x^3 + 4x^2 - 5x + 5}$$

2. Write c for the divisor, $x - c$. To the right, write the coefficients of the dividend.

$$\underline{3|}\quad 1 \quad 4 \quad -5 \quad 5$$

3. Write the leading coefficient of the dividend on the bottom row.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ & \downarrow & \text{Bring down 1.} & & \\ \hline & 1 & & & \end{array}$$

4. Multiply c (in this case, 3) times the value just written on the bottom row. Write the product in the next column in the second row.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ & & 3 & & \\ \hline & 1 & & & \end{array}$$

Multiply by 3: $3 \cdot 1 = 3$.

5. Add the values in this new column, writing the sum in the bottom row.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ & & 3 & & \\ \hline & 1 & 7 & & \end{array}$$

Add.

6. Repeat this series of multiplications and additions until all columns are filled in.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ & & 3 & 21 & \\ \hline & 1 & 7 & 16 & \end{array}$$

Add.

Multiply by 3: $3 \cdot 7 = 21$.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -5 & 5 \\ & & 3 & 21 & 48 \\ \hline & 1 & 7 & 16 & 53 \end{array}$$

Add.

Multiply by 3: $3 \cdot 16 = 48$.

7. Use the numbers in the last row to write the quotient, plus the remainder above the divisor. **The degree of the first term of the quotient is one less than the degree of the first term of the dividend.** The final value in this row is the remainder.

Written from
1 7 16 53
the last row of the synthetic division

$$\begin{array}{r} 1x^2 + 7x + 16 + \dfrac{53}{x - 3} \\ x - 3\overline{)x^3 + 4x^2 - 5x + 5} \end{array}$$

EXAMPLE 1 Using Synthetic Division

Use synthetic division to divide $5x^3 + 6x + 8$ by $x + 2$.

SOLUTION The divisor must be in the form $x - c$. Thus, we write $x + 2$ as $x - (-2)$. This means that $c = -2$. Writing a 0 coefficient for the missing x^2-term in the dividend, we can express the division as follows:

$$x - (-2)\overline{)5x^3 + 0x^2 + 6x + 8}.$$

Now we are ready to set up the problem so that we can use synthetic division.

Use the coefficients of the dividend $5x^3 + 0x^2 + 6x + 8$ in descending powers of x.

This is c in $x - (-2)$.

$$\underline{-2}|\quad 5 \quad 0 \quad 6 \quad 8$$

We begin the synthetic division process by bringing down 5. This is followed by a series of multiplications and additions.

1. Bring down 5.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & & & \\ \hline & 5 & & & \end{array}$$

2. Multiply: $-2(5) = -10$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & & \\ \hline & 5 & & & \end{array}$$

Multiply by -2.

3. Add: $0 + (-10) = -10$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & & \\ \hline & 5 & -10 & & \end{array}$$

Add.

4. Multiply: $-2(-10) = 20$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & \\ \hline & 5 & -10 & & \end{array}$$

Multiply by -2.

5. Add: $6 + 20 = 26$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & \\ \hline & 5 & -10 & 26 & \end{array}$$

Add.

6. Multiply: $-2(26) = -52$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & \end{array}$$

Multiply by -2.

7. Add: $8 + (-52) = -44$.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

Add.

The numbers in the last row represent the coefficients of the quotient and the remainder. The degree of the first term of the quotient is one less than that of the dividend. Because the degree of the dividend, $5x^3 + 6x + 8$, is 3, the degree of the quotient is 2. This means that the 5 in the last row represents $5x^2$.

Thus,

$$\begin{array}{r} 5x^2 - 10x + 26 - \dfrac{44}{x+2} \\ x + 2\overline{)5x^3 + \quad 6x + \quad 8} \end{array}$$

Practice:

1, $\dfrac{x^2-6x+5}{x-1}$ Answer: $x-5$

2. $\dfrac{2x^2+8x+13}{x+2}$ Answer: $2x+4+\dfrac{5}{x+2}$

3, $(5x^3-6x^2+14)\div(x+1)$ Answer: $5x^2-11x+11+\dfrac{3}{x+1}$

Practice:

1, $\dfrac{x^2-6x+5}{x-1}$ Answer: $x-5$

1 −6 5

$$\begin{array}{r|rrr} +1 & 1 & -6 & 5 \\ & \downarrow & 1 & -5 \\ \hline & 1 & -5 & 0 \end{array}$$

$1 \cdot x - 5 \quad R(0)$

2. $\dfrac{2x^2+8x+13}{x+2}$ Answer: $2x+4+\dfrac{5}{x+2}$

2 8 13

−2

$$\begin{array}{r|rrr} -2 & 2 & 8 & 13 \\ & \downarrow & -4 & -8 \\ \hline & 2 & 4 & 5 \end{array}$$

$2x+4 \quad R(5)$

3, $(5x^3-6x^2+14)\div(x+1)$ Answer: $5x^2-11x+11+\dfrac{3}{x+1}$

no x replace by 0

5 −6 0 14 −1

$$\begin{array}{r|rrrr} -1 & 5 & -6 & 0 & 14 \\ & \downarrow & -5 & 11 & -11 \\ \hline & 5 & -11 & 11 & 3 \end{array}$$

$5x^2-11x+11 \quad R(3)$