MAT150 - Sequence/Sum Review
Chapter 7
Spring 2002 Name:________________________________

1. Determine the first 4 terms and the 10th term of: $a_n = \dfrac{n}{n^2+2}$

First 4 terms:

____

10th term:__________

2. The sequence is defined recursively: Find the first 5 terms

$a_1 = 2 ; \quad a_n = n + a_{n-1}$ ______________________________

3. Calculate the following sum: 2.________________

$$\sum_{k=3}^{5} 2k^2$$

4. Using the Propert ... s write out each sum --
... JUST SUBSTITUTE

SKIP

$$\sum_{k=1}^{n} \left(k^3 - 2k + \right.$$

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6. For the geometric sequence, determine the common ratio and a formula for the nth term. (a_n).

1, -2, 4, -8, 16, ... Common Ratio: __________________

nth term (a_n):__________________________

7. Find the sums of the following Geometric Sequences:

a. $\sum_{k=1}^{12} \frac{1}{16}\left(2^k\right)$

b. $\sum_{k=1}^{20} 2(1.2)^{k-1}$

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c. $\sum_{k=1}^{\infty} 3\left(\frac{3}{2}\right)^{k-1}$

d. $\sum_{k=1}^{\infty} 4\left(\frac{1}{2}\right)^{k}$

e, $\sum_{k=1}^{\infty} 3(.1)^{k-1}$

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1. Determine the first 4 terms and the 10th term of: $a_n = \frac{n}{n^2+2}$

First 4 terms: $\frac{1}{3}, \frac{1}{3}, \frac{3}{11}, \frac{2}{9}$

10th term: $\frac{5}{51}$

$a_1 = \frac{1}{1^2+2} = \frac{1}{3}$

$a_2 = \frac{2}{2^2+2} = \frac{2}{6} = \frac{1}{3}$

$a_3 = \frac{3}{3^2+2} = \frac{3}{11}$

$a_4 = \frac{4}{4^2+2} = \frac{4}{18} = \frac{2}{9}$

$a_{10} = \frac{10}{10^2+2} = \frac{10}{102} = \frac{5}{51}$

2. The sequence is defined recursively: Find the first 5 terms
$a_1 = 2$; $a_n = n + a_{n-1}$

2, 4, 7, 11, 16

$a_2 = 2 + a_{2-1} = 2 + a_1 = 2 + 2 = 4$

$a_3 = 3 + a_{3-1} = 3 + a_2 = 3 + 4 = 7$

$a_4 = 4 + a_{4-1} = 4 + a_3 = 4 + 7 = 11$

$a_5 = 5 + a_{5-1} = 5 + a_4 = 5 + 11 = 16$

3. Calculate the following sum:

2. 100

$$\sum_{k=3}^{5} 2k^2 = 2(3)^2 + 2(4)^2 + 2(5)^2$$
$$= 2(9) + 2(16) + 2(25)$$
$$= 100$$

4. Using the Properties of Sequences/Sums write out each sum --
DO NOT SIMPLIFY-- JUST SUBSTITUTE

$$\sum_{k=1}^{n}\left(k^3 - 2k + 3\right)$$

$$\sum_{k=1}^{n} k^3 - 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

skip

$$\boxed{\frac{n^2(n+1)^2}{4} - 2\left[\frac{n(n+1)}{2}\right] + 3n}$$

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4. For the arithmetic sequence determine the common difference and a formula for the nth term (a_n) Using the formula you found for a_n, find a_{20}:

-5, -2, 1, 4, … Common Difference: 3

$-2 - -5 = 3 = d$

$a_1 = -5$

nth term (a_n): $a_n = 3n-8$

$a_{20} =$ 52

$a_n = -5 + (n-1)(3)$
$= -5 + 3n - 3$
$= 3n - 8$

$a_{20} = 3(20) - 8$
$= 60 - 8$
$= 52$

5. Find the following arithmetic sums:

a. 1 + 5 + 9 + 13 + …+ (4n - 3) $n = n$ — $2n^2 - n$

$\underset{1}{} \longrightarrow n \rightarrow a_1 = 1$

$a_n = 4n-3$

$S_n = \frac{n}{2}[1 + 4n - 3] = \frac{n}{2}[4n-2) = n(2n-1)$

b. 3 + 9 + 15 + 21 + … + 117 $d = 6$ — 1200

$a_1 = 3$
$a_n = 117$
need n?
20

$117 = 3 + (n-1)6$
$114 = 6n - 6$ $n = 20$
$120 = 6n$

$S_{20} = \frac{20}{2}[3 + 117] = 10[120]$
$= 1200$

c. $\sum_{k=1}^{50}(2k+1)$ $n = 50$ — 2600

$k = 1 \Rightarrow a_1 = 2(1) + 1 = 3$

$k = 50 \Rightarrow a_{50} = 2(50) + 1 = 101$

$S_{50} = \frac{50}{2}[a_1 + a_{50}]$
$= 25[3 + 101]$
$= 25[104] = 2600$

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6. For the geometric sequence, determine the common ratio and a formula for the nth term. (a_n).

1, -2, 4, -8, 16, … Common Ratio: $r = -2$

nth term (a_n): $a_n = (-2)^{n-1}$

$\frac{4}{-2} = -2 = r$, $a_1 = 1$

$a_n = a_1(r)^{n-1}$

$a_n = 1 \cdot (-2)^{n-1}$

7. Find the sums of the following Geometric Sequences: $S_n = \frac{a_1(1-r^n)}{1-r}$

a. $\sum_{k=1}^{12} \frac{1}{16}(2^k)$

$n = 12$

$a_1 = \frac{1}{16}(2^1) = \frac{1}{8}$

$r = 2$

$S_{12} = \frac{\frac{1}{8}(1-2^{12})}{1-2} = \frac{-511.9}{-1} = 511.9$

b. $\sum_{k=1}^{20} 2(1.2)^{k-1}$

$n = 20$

$a_1 = 2(1.2)^{1-1} = 2(1.1)^0 = 2$

$r = 1.2$

$S_{20} = \frac{2(1-1.2^{20})}{1-1.2} = \frac{-74.7}{-.2} = 373.4$

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c. $\sum_{k=1}^{\infty} 3\left(\frac{3}{2}\right)^{k-1}$

$S_\infty = \frac{a_1}{1-r}$ $|r| < 1$ $a_1 = 3\left(\frac{3}{2}\right)^{1-1} = 3$

$r = \frac{3}{2} > 1$

so S_∞ does not exist

d. $\sum_{k=1}^{\infty} 4\left(\frac{1}{2}\right)^{k}$

$S_\infty = \frac{a_1}{1-r}$ $a_1 = 4\left(\frac{1}{2}\right)^1 = 2$

$r = \frac{1}{2} < 1$ so S_∞ exists

$S_\infty = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$

$\boxed{S_\infty = 4}$

e, $\sum_{k=1}^{\infty} 3(.1)^{k-1}$

$a_1 = 3(.1)^{1-1} = 3$

$S_\infty = \frac{a_1}{1-r}$ $r = .1 < 1$ so S_∞ exists

$S_\infty = \frac{3}{1-.1} = \frac{3}{.9} = \frac{30}{9} = 3\frac{1}{3}$

$S_\infty = 3\frac{1}{3}$