mat 150 - chapter 4 review Key Spring 2010

(1) $f(x) = \dfrac{x}{x+1}$; $x \neq -1$

$y = \dfrac{x}{x+1}$

$x = \dfrac{y}{y+1}$

$(y+1)x = y$

$xy + x = y$

$x = y - xy$

$x = y(1-x)$

$\dfrac{x}{1-x} = y$

$\boxed{f^{-1}(x) = \dfrac{x}{1-x}}$ $x \neq 1$

$D_f: \mathbb{R}, x \neq -1$ $D_{f^{-1}}: \mathbb{R}; x \neq 1$ (1)

$R_f: \mathbb{R}; y \neq 1$ $R_{f^{-1}}: \mathbb{R}; y \neq -1$

Verify $f(f^{-1}(x)) = x$

$f\left(\dfrac{x}{1-x}\right) = \left[\dfrac{\left(\frac{x}{1-x}\right)}{\left(\frac{x}{1-x}\right) + 1}\right] \dfrac{\frac{1-x}{1}}{\frac{1-x}{1}}$

$= \dfrac{x}{x + 1(1-x)}$

$= \dfrac{x}{x+1-x} = x$ ✓

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(2) $\dfrac{2x-1}{(x+2)^2} > 0$

Domain: $\left(\frac{1}{2}, \infty\right)$

(=) $2x - 1 = 0$

$x = \frac{1}{2}$

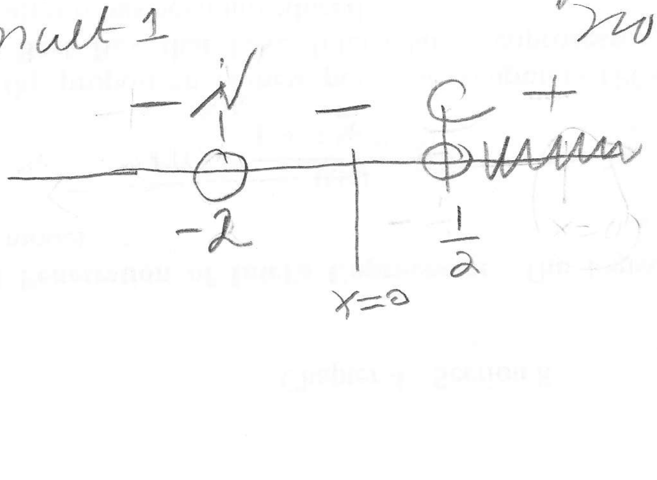
mult 1

(≠) $(x+2)^2 \neq 0$

$x \neq -2$

mult 2

no change



$\dfrac{2(0)-1}{(0+2)^2} = \dfrac{-1}{4} < 0$

3 a) $5^{2x} = 3y \Rightarrow \log_5 3y = 2x$

b) $\log_b 5y = 2a \Rightarrow b^{2a} = 5y$

4 a) $\ln\left(\frac{2}{3x^3y^2}\right)$

$\ln(2) - \ln(3x^3y^2)$

$\ln(2) - [\ln 3 + \ln x^3 + \ln y^2]$

$\ln(2) - [\ln 3 + 3\ln x + 2\ln y]$ ⇐ this ok too –

$\boxed{\ln 2 - \ln 3 - 3\ln x - 2\ln y}$

b) $\log_a \sqrt[3]{\frac{y^2z^3}{x^3}} = \log_a\left(\frac{y^2z^3}{x^3}\right)^{1/3}$

$\frac{1}{3}\log_a\left(\frac{y^2z^3}{x^3}\right)$

$\frac{1}{3}[\log_a y^2z^3 - \log_a x^3]$

$\frac{1}{3}[\log_a y^2 + \log_a z^3 - \log_a x^3]$

$\frac{1}{3}[2\log_a y + 3\log_a z - 3\log_a x]$ ⇐ this ok too

$\boxed{\frac{2}{3}\log_a y + \log_a z - \log_a x}$

5 a) $5\log_b x + \frac{1}{2}\log_b y - 4\log_b z$

$\log_b x^5 + \log_b y^{\frac{1}{2}} - \log_b z^4$

$\log_b (x^5 y^{\frac{1}{2}}) - \log_b z^4$

$\log_b \left[\frac{x^5 y^{\frac{1}{2}}}{z^4}\right]$

6. a) $5^{x^2-2x} = 5^{4x}$

$x^2 - 2x = 4x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \quad x = 6$

b) $9^{2x-5} = 27^{4x+1}$

$(3^2)^{2x-5} = (3^3)^{4x+1}$

$3^{2(2x-5)} = 3^{3(4x+1)}$

$2(2x-5) = 3(4x+1)$

$4x - 10 = 12x + 3$

$-10 = 8x + 3$

$-13 = 8x$

$\frac{-13}{8} = x$

c) $e^{3x-1} = 2$

$\ln(e^{3x-1}) = \ln 2$

$(3x-1)\cancel{\ln e}^{1} = \ln 2$

$3x - 1 = \ln 2$

$3x = 1 + \ln 2$

$x = \frac{1+\ln 2}{3}$

d) $3^{2x-1} = 5$

$\log(3^{2x-1}) = \log 5$

$(2x-1)\log 3 = \log 5$

$2x\log 3 - \log 3 = \log 5$

$2x\log 3 = \log 5 + \log 3$

$x = \frac{\log 5 + \log 3}{2\log 3}$

7a) $\ln(5x+8) = 4$

Domain:

$5x+8 > 0$

$5x > -8$

$x > -\frac{8}{5}$

Solution:

$\ln(5x+8) = 4$

$\log_e(5x+8) = 4$

$5x+8 = e^4$

$5x = e^4 - 8$

$x = \frac{e^4-8}{5}$

b) $\log_{(x+2)} 81 = 2$

Domain

Base must be +

$x+2 > 0$

$x > -2$

Solution

$(x+2)^2 = 81$

$x+2 = \pm\sqrt{81}$

$x+2 = \pm 9$

$x = -2 \pm 9$

$x = -2+9 = 7$

$x = -2-9 = \cancel{-11}$

$x = 7$

c) $\log_4(x+3) + \log_4(x-3) = 2$

Domain:

$x+3 > 0$ and $x-3 > 0$

$x > -3$ and $x > 3$

Bigger of 2: $x > 3$

Solution:

$\log_4(x+3) + \log_4(x-3) = 2$

$\log_4(x+3)(x-3) = 2$

$(x+3)(x-3) = 4^2$

$x^2 - 9 = 16$

$x^2 = 25$

$x = \pm 5$

$x \neq -5$ so $x = 5$

7 d) $\log(x+3) - \log(x-2) = 2$

Domain:

$x+3>0$ and $x-2>0$

$x>-3$ and $x>2$

bigger of 2 is $x>2$

Solution

$\log\left[\frac{x+3}{x-2}\right] = 2$

base 10

$$\frac{x+3}{x-2} = 10^2$$

$$\frac{x+3}{x-2} = 100$$

$$x+3 = 100(x-2)$$

$$x+3 = 100x - 200$$

$$3 = 99x - 200$$

$$203 = 99x$$

$$\frac{203}{99} = x$$

8. $3^{2x} - 3^{x} - 6 = 0$

use substitution
$u = 3^{x}$
$u^2 = 3^{2x}$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3 \quad u = -2$

$3^{x} = 3^{1} \quad 3^{x} = -2$

$x = 1$ — Can't happen

$(\log_2(x+4))^2 - 4\log_2(x+4) + 3 = 0$

use substitution
$u = \log_2(x+4)$
$u^2 = (\log_2(x+4))^2$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$u = 3 \quad u = 1$

$\log_2(x+4) = 3 \quad \log_2(x+4) = 1$

$x+4 = 2^3 \quad x+4 = 2^1$

$x+4 = 8 \quad x+4 = 2$

$x = 4 \quad x = -2$