NAME Key

SCORE__________/ 85

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NO CALCULATORS ON THIS SECTION EXCEPT FOR ARITHMETIC CALCULATIONS – ALL WORK MUST BE SHOWN TO RECEIVE CREDIT!!!!

1. Given the following matrix:

$$\begin{bmatrix} 1 & 0 \\ -3 & 1/2 \\ 4 & 1 \\ 2/3 & -1 \end{bmatrix}$$

(rows labelled 1, 2, 3, 4; columns labelled 1, 2) 4×2

a. Determine the order (size) of the matrix: a. 4×2 (2)

2. Write the **augmented** matrix for the following system:

$$5x - 3y = 1$$
$$-4x = -2$$

2. $\left[\begin{array}{cc|c} 5 & -3 & 1 \\ -4 & 0 & -2 \end{array}\right]$ (3)

3. Given the following matrix in upper triangular form, solve the system for x, y and z.

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & -2 \end{array}\right]$$

x= −4 y= 2 z= −1 (4)

$x + 2y - 2z = 2$

$y + 3z = -1$

$2z = -2$

$z = -1$

$y + 3(-1) = -1$

$y = 2$

$x + 2(2) - 2(-1) = 2$

$x + 4 + 2 = 2$

$x + 6 = 2$

$x = -4$

4. Using **matrix** methods, solve the following system. **If you use your calculator to do the problem, list the operations you used to get the solution, otherwise show ALL work.** (8 points) Write your answer as an (x,y,z) coordinate point.

NOTE: rref is NOT an acceptable calculator method to solve this problem you must use matrix row operations!!!!

$$\begin{aligned} x - 2y - z &= 1 \\ -x + 3y + 3z &= 4 \\ 2x - 3y + z &= 10 \end{aligned}$$

Solution: $(2, -1, 3)$

$r_1 + r_2 = R_2$
$-2r_1 + r_3 = R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ -1 & 3 & 3 & 4 \\ 2 & -3 & 1 & 10 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 3 & 8 \end{array}\right] \quad -1r_2 + r_3$$

$$\begin{array}{rcccc} -2r_1 = & -2 & 4 & 2 & -2 \\ +r_3 & 2 & -3 & 1 & 10 \\ \hline R_3 & 0 & 1 & 3 & 8 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$x - 2y - z = 1$$
$$y + 2z = 5$$
$$z = 3$$
$$y + 2(3) = 5$$
$$y + 6 = 5$$
$$y = -1$$
$$x - 2(-1) - 3 = 1$$
$$x + 2 - 3 = 1$$
$$x - 1 = 1$$
$$x = 2$$

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5. Find the values of a, b, and c such that the graph of the quadratic equation $y = ax^2 + bx + c$ passes through the points (–1, –7), (2, 2), and (3, 1). (8 points)

Set up the system and solve it. Use your calculator and the RREF command for this one. Write your answer as $y = ax^2 + bx + c$.

$y = a x^2 + b x + c$

$-7 = a(-1)^2 + b(-1) + c$

$2 = a(2)^2 + b(2) + c$

$1 = a(3)^2 + b(3) + c$

$a - b + c = -7$

$4a + 2b + c = 2$

$9a + 3b + c = 1$

$a = -1$

$b = 4$

$c = -2$

so $y = -x^2 + 4x - 2$

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6. Solve the following non-linear system: $\begin{cases} xy = 1 \\ y = 2x + 1 \end{cases}$ (8)

State your answers as coordinate points.

substitute

$(\frac{1}{2}, 2)$

$(-1, -1)$

$x(2x+1) = 1$

$2x^2 + x = 1$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = \frac{1}{2}$ $\quad x = -1$

$y = 2(\frac{1}{2}) + 1$ $\quad y = 2(-1) + 1$

$y = 2$ $\quad y = -1$

7. Solve the following non-linear system: $\begin{cases} x^2 + y^2 = 6 \\ 3x^2 - y^2 = 10 \end{cases}$ (8)

State your answers as coordinate points.

$(2, \pm\sqrt{2})$

$(-2, \pm\sqrt{2})$

add

$4x^2 = 16$

$x^2 = 4$

$x = \pm 2$

$x = 2$

$y^2 = 6 - x^2$

$y^2 = 6 - 2^2$

$y^2 = 2$

$y = \pm\sqrt{2}$

$x = -2$

$y^2 = 6 - (-2)^2$

$y^2 = 6 - 4$

$y^2 = 2$

$y = \pm\sqrt{2}$

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8. Identify the following conic sections: (2 points each)

i) $4x^2 + 2y^2 = 16$ i) C

a) Circle b) Parabola c) Ellipse d) Hyperbola

ii) $6x^2 + 8x - y = 20$ ii) b

a) Circle b) Parabola c) Ellipse d) Hyperbola

iii) $16x^2 - 16y^2 = 32$ iii) d

a) Circle b) Parabola c) Ellipse d) Hyperbola

iv) $9x^2 + 9y^2 = 36$ iv) a

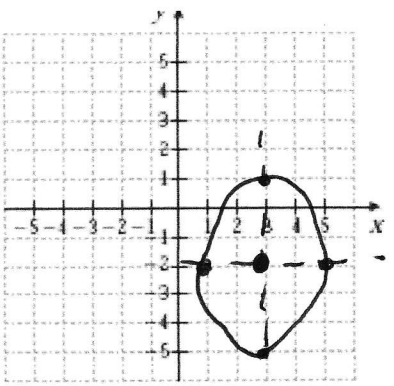
a) Circle b) Parabola c) Ellipse d) Hyperbola

9. Graph the following Conic Sections:

a) $\frac{(x-3)^2}{4} + \frac{(y+2)^2}{9} = 1$ ellipse (5 points)

3 -2

±2 ±3

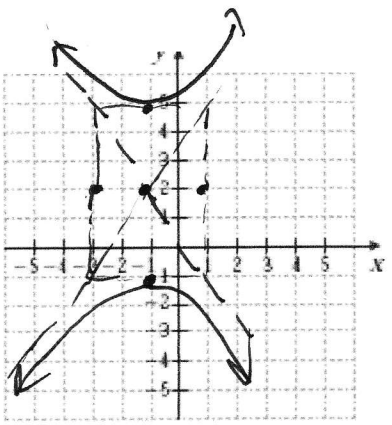


b) $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{4} = 1$ hyperbola (5 points)

2 -1

±3 ±2

on y on x



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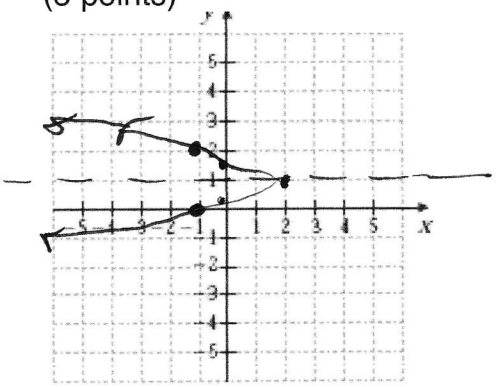
parabola

c) $x = -3(y-1)^2 + 2$ (3 points)

Opens: left (1)

Vertex: (2, 1) (2)

Axis of Symmetry: $y = 1$ (1)



x-Intercept(s): $(-1, 0)$ (3)

$y = 0$

$x = -3(0-1)^2 + 2$

$= -3 + 2$

$= -1$

y-Intercept(s): $(0, 1.8)\ (0, .18)$ (3)

$x = 0$

$0 = -3(y-1)^2 + 2$

$-2 = -3(y-1)^2$

$\frac{2}{3} = (y-1)^2$

$1 \pm \sqrt{\frac{2}{3}} = y$

$1 + \sqrt{\frac{2}{3}}$ $\quad 1 - \sqrt{\frac{2}{3}}$

1.8 $\quad .18$

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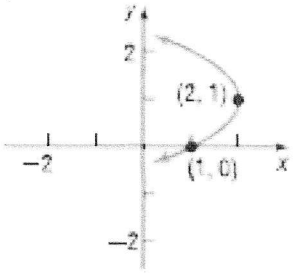
10. Given the following graphs find the equation of the conic sections in standard form.

a)



$X = a(y-k)^2 + h$

$X = a(y-1)^2 + 2$ if $x=1$ $y=0$

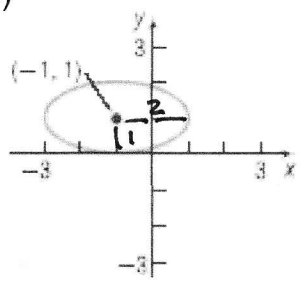
$1 = a(0-1)^2 + 2$

$-1 = a$

$X = -1(y-1)^2 + 2$

a) $X = -1(y-1)^2 + 2$ (5)

b)

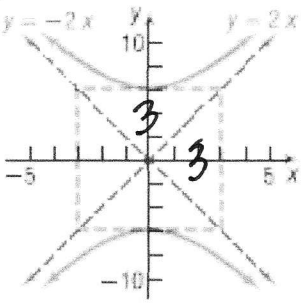


$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ ($x-h$: $x-(-1)$; $y-k$: $y-1$; $a^2 = 2^2$; $b^2 = 1^2$)

$\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$

b) $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$ (4)

c)



$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ ($y-k$: $y-0$; $a^2 = 3^2$; $b^2 = 3^2$)

$\frac{y^2}{9} - \frac{x^2}{9} = 1$

c) $\frac{y^2}{9} - \frac{x^2}{9} = 1$ (4)