

1. Given $f(x) = \frac{4x}{x-5}$

Domain of f : $\mathbb{R}; x \neq 5$ (1)

Domain of f^{-1} : $\mathbb{R}; x \neq 4$ (1)

Range of f : $\mathbb{R}; y \neq 4$ (1)

Range of f^{-1} : $\mathbb{R}; y \neq 5$ (1)

a. Find a formula for the inverse -- (4 points)

$f^{-1}(x) = \frac{5x}{x-4}$ (4)

$y = \frac{4x}{x-5}$

$x = \frac{4y}{y-5}$

$x(y-5) = 4y$

$xy - 5x = 4y$

$xy - 4y = 5x$

$y(x-4) = 5x$

$y = \frac{5x}{x-4}$

↑
Domain
 $\mathbb{R}; x \neq 4$

2. a. Find the domain of: $f(x) = \log_5\left(\frac{2x}{3x+1}\right)$

argument > 0

$\frac{2x}{3x+1} > 0$

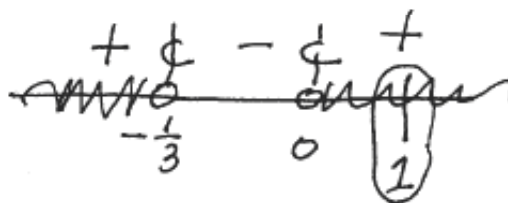
a. $(-\infty, -\frac{1}{3}) \cup (0, \infty)$ (5)

$\Leftrightarrow \frac{2x}{3x+1} = 0$

$2x = 0$

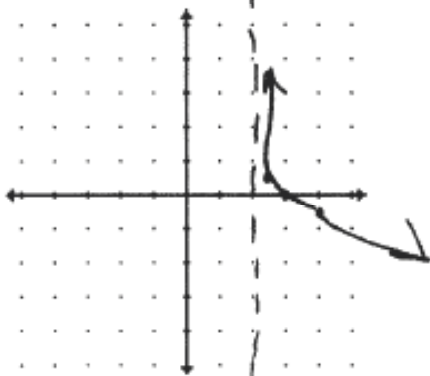
$x = 0$

$\Leftrightarrow 3x+1 \neq 0$
 $x \neq -\frac{1}{3}$



3. Graph the following function using transformations. Label at least 3 coordinate points on your graph along with any vertical or horizontal asymptotes: (3 points)

a. $f(x) = -\frac{1}{2} \ln(x-2)$



+2 x	ln x	* -1/2
2 0	VA	VA
2 1/2 1/2	-0.7	-0.7/2 ≈ -0.4
3 1	0	0 = 0
4 2	0.7	-0.7/2 ≈ -0.4

List the Transformation: (4 points)

- 1: Basic Function: $y = \ln x$
- 2: right 2
- 3: reflect x-axis
- 4: v. shrink factor of $\frac{1}{2}$

4. Write in exponential notation: $\log_2 3a = y$

4.5. $2^y = 3a$

5. Write in logarithmic notation: $5^{(7a)} = 2b$

5.8. $\log_5 2b = 7a$

6. Write as a single log:

$\frac{1}{2} \log_b x + 5 \log_b y - 3 \log_b z$ (5)

$$\log_b x^{\frac{1}{2}} + \log_b y^5 - \log_b z^3$$

$$\log_b (x^{\frac{1}{2}} y^5) - \log_b z^3$$

$$\log_b \left[\frac{x^{\frac{1}{2}} y^5}{z^3} \right]$$

7. Write as a sum, difference or product of logs. $\log \frac{\sqrt[3]{x^2 y^2}}{z^5}$ (5)

$$\begin{aligned} & \log (x^2 y^2)^{\frac{1}{3}} - \log z^5 \\ & \frac{1}{3} \log (x^2 y^2) - 5 \log z \\ & \frac{1}{3} [\log x^2 + \log y^2] - 5 \log z \\ & \frac{1}{3} [2 \log x + 2 \log y] - 5 \log z \end{aligned}$$

8. Solve the following equations **ALGEBRAICALLY** for x. **SHOW ALL WORK.**

a. $5^{x^2-x} = 5^{3x}$ a. $x=0 \quad x=4$ (4)

$$\begin{aligned} x^2 - x &= 3x \\ x^2 - 4x &= 0 \\ x(x-4) &= 0 \\ x=0 \quad x &= 4 \end{aligned}$$

b. $8^{x-5} = (2)^{2x-1}$ b. $x=14$ (4)

$$\begin{aligned} (2^3)^{x-5} &= 2^{2x-1} \\ 3(x-5) &= 2x-1 \\ 3x-15 &= 2x-1 \\ x-15 &= -1 \\ x &= 14 \end{aligned}$$

c. $e^{2x+3} = 5$ c. $x = \frac{-3 + \ln 5}{2}$ (4)

$$\begin{aligned} \ln e^{2x+3} &= \ln 5 \\ (2x+3) \ln e &= \ln 5 \\ 2x+3 &= \ln 5 \\ 2x &= -3 + \ln 5 \\ x &= \frac{-3 + \ln 5}{2} \end{aligned}$$

d. $\log_{(x+5)} 36 = 2$

$$(x+5)^2 = 36$$

$$x+5 = \pm\sqrt{36}$$

$$x = -5 \pm 6$$

$$x = -5 - 6$$

$$x = \cancel{-11}$$

$$x = -5 + 6$$

$$x = 1$$

d. $x = 1$ (4)

e. $\ln(5x+2) = 3$

$$\log_e(5x+2) = 3$$

$$5x+2 = e^3$$

$$5x = e^3 - 2$$

$$x = \frac{e^3 - 2}{5}$$

e. $x = \frac{e^3 - 2}{5}$ (4)

f. $\ln(x) + \ln(x+5) = \ln(4x+6)$

$$\ln x(x+5) = \ln(4x+6)$$

$$x(x+5) = 4x+6$$

$$x^2 + 5x = 4x + 6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = \cancel{3} \quad x = 2$$

d. $x = 2$ (5)

g. $\log_4(x+4) - \log_4(x-3) = 2$

$$\log_4 \left[\frac{x+4}{x-3} \right] = 2$$

$$\frac{x+4}{x-3} = 4^2$$

$$\frac{x+4}{x-3} = 16$$

$$x+4 = 16(x-3)$$

$$x+4 = 16x - 48$$

Domain:
 $x+4 > 0 \wedge x-3 > 0$
 $x > -4 \wedge x > 3$

g. State the domain of the solution set:

 $x > 3$ (2)

Solution: $x = \frac{52}{15}$ (5)

$$-15x = -52$$

$$x = \frac{52}{15}$$

h. $\log(x) + \log(x+15) = 2$

$$\log_{10} x(x+15) = 2$$

$$x(x+15) = 10^2$$

$$x^2 + 15x = 100$$

$$x^2 + 15x - 100 = 0$$

$$(x+20)(x-5) = 0$$

$$x = \cancel{0} \quad x = 5$$

Solution: $x = 5$ (5)

i) $3^{2x} - 3^x - 72 = 0$

$$u = 3^x$$

$$u^2 = (3^x)^2 = 3^{2x}$$

$$u^2 - u - 72 = 0$$

$$(u-9)(u+8) = 0$$

$$u = 9 \quad u = -8$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x = 2$$

~~$$3^x = -8$$~~

i. $x = 2$ (5)