**DEFINITION OF THE EXPONENTIAL FUNCTION** **The exponential function $f$ with base $b$ is defined by**

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

**where $b$ is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and $x$ is any real number.**

Examples: $f(x) = 2^x$ $g(x) = 4^x$ $h(x) = 5^{x+1}$

Evaluating an exponential function:

$f(x) = 2^x$ Find: f(3), f(–3), f(.5)

$f(3) = 2^3 = 8$ $f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $f(.5) = 2^{.5} = 2^{1/2} = \sqrt{2} \approx 1.414$

**Using a Calculator:**

Scientific Calculator: f(1.5) = 2 $\mathbf{x^y}$ 1.5
Graphing Calculator: : f(1.5) = 2 ^ 1.5

**A special number**: $e \approx 2.718281827...$ --- you can approximate it to 2.7 or 2.72 for calculation.

*In Exercises 1–10, approximate each number using a calculator. Round your answer to three decimal places.*

**2.** $3^{2.4} = 13.967$

**4.** $5^{\sqrt{3}} = 16.242$

**6.** $6^{-1.2} = .116$

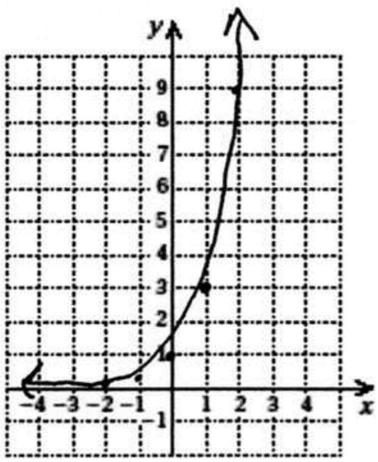
**8.** $e^{3.4} = 29.964$

**10.** $e^{-0.75} = .472$

Graphing an exponential function:

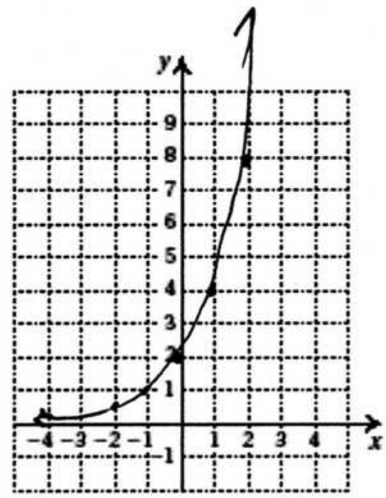
**2.** $f(x) = 3^x$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $3^{-2} = \frac{1}{9}$ |
| $-1$ | $3^{-1} = \frac{1}{3}$ |
| $0$ | $3^0 = 1$ |
| $1$ | $3^1 = 3$ |
| $2$ | $3^2 = 9$ |
| $3$ | $3^3 = 27$ |



$f(x) = 2^{x+1}$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $\frac{1}{2}$ |
| $-1$ | $1$ |
| $0$ | $2$ |
| $1$ | $4$ |
| $2$ | $8$ |
| $3$ | $16$ |



$f(-2) = 2^{-2+1} = 2^{-1} = \frac{1}{2}$

$f(-1) = 2^{-1+1} = 2^0 = 1$

$f(0) = 2^{0+1} = 2^1 = 2$

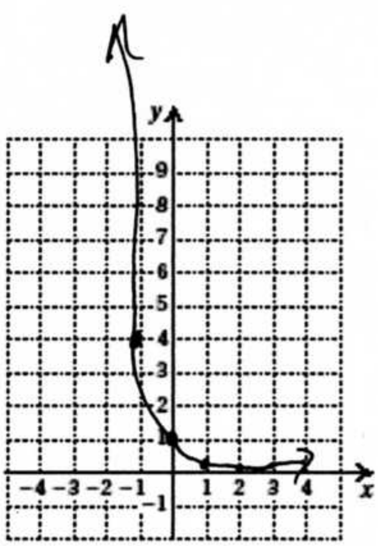
$f(1) = 2^{1+1} = 2^2 = 4$

$f(2) = 2^{2+1} = 2^3 = 8$

$f(3) = 2^{3+1} = 2^4 = 16$

$f(x) = \left(\frac{1}{4}\right)^x$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $16$ |
| $-1$ | $4$ |
| $0$ | $1$ |
| $1$ | $\frac{1}{4}$ |
| $2$ | $\frac{1}{16}$ |
| $3$ | $\frac{1}{64}$ |



$f(-2) = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$

$f(-1) = \left(\frac{1}{4}\right)^{-1} = 4$

$f(0) = \left(\frac{1}{4}\right)^0 = 1$

$f(1) = \left(\frac{1}{4}\right)^1 = \frac{1}{4}$

$f(2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$f(3) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

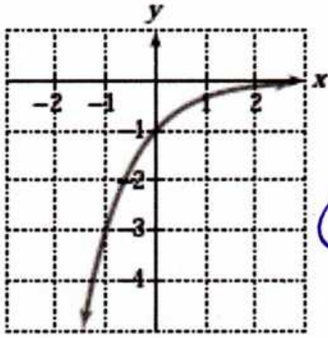
← in class do evens

**11.** $f(x) = 3^x$  **12.** $f(x) = 3^{x-1}$

**13.** $f(x) = 3^x - 1$  **14.** $f(x) = -3^x$

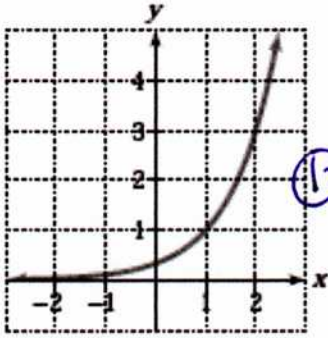
**15.** $f(x) = 3^{-x}$  **16.** $f(x) = -3^{-x}$

**(a)**



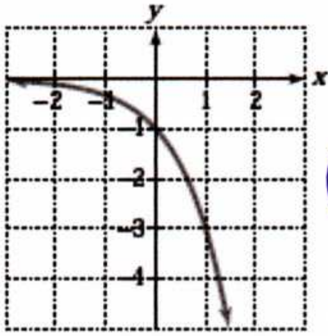

16

**(b)**



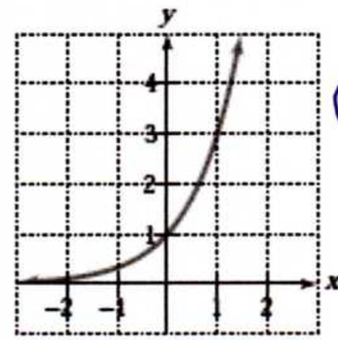

12

**(c)**




14

**(d)**

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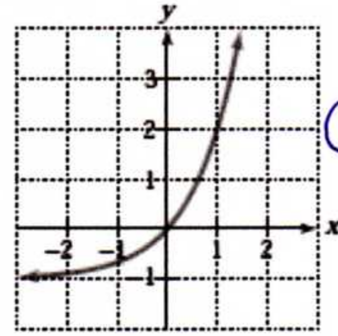
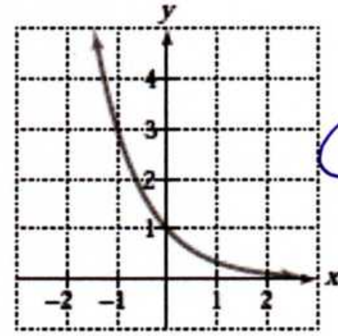
**(e)**




13

**(f)**




15

11! $f(x) = 3^x$
$f(1) = 3^1 = 3$

12. $f(x) = 3^{x-1}$
$f(1) = 3^{1-1} = 3^0 = 1$

13 $f(x) = 3^x - 1$
$f(1) = 3^1 - 1 = 2$

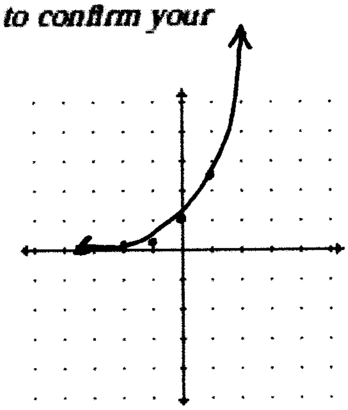
14. $f(x) = -3^x$
$f(1) = -3^1 = -3$

15. $f(x) = 3^{-x}$
$f(1) = 3^{-1} = \frac{1}{3}$

16 $f(x) = -3^{-x}$
$f(1) = -3^{-1} = -\frac{1}{3}$

***In Exercises 17–24, graph each function by making a table of coordinates. If applicable, use a graphing utility to confirm your hand-drawn graph.***

26. $f(x) = e^x$



| $x$ | $e^x$ |
|---|---|
| $-2$ | $e^{-2} = .13$ |
| $-1$ | $e^{-1} = .37$ |
| $0$ | $e^0 = 1$ |
| $1$ | $e^1 = 2.7$ |
| $2$ | $e^2 = 7.4$ |

*Use the compound interest formulas* $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *and* $A = Pe^{rt}$*, to solve Exercises 39–42. Round answers to the nearest cent.*

P t

42. Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is **a.** compounded semiannually; **b.** compounded monthly; **c.** compounded continuously.

r

a) compounded semiannually → $n=2$

$P=5000 \quad t=10 \quad r=.065 \quad n=2$

$A = 5000\left(1+\frac{.065}{2}\right)^{2(10)}$

$= 5000(1.0325)^{20}$

$= \$9479.19$

b) Compounded monthly → $n=12$

$P=5000 \quad t=10 \quad r=.065 \quad n=12$

$A = 5000\left(1+\frac{.065}{12}\right)^{12(10)}$

$= 5000(1.0054)^{120}$

$= \$9541.92$

c) Compounded continuously

$P=5000 \quad t=10 \quad r=.065$

$A = 5000e^{.065(10)}$

$= \$9577.70$

$P$

44. Suppose that you have $6000 to invest. Which investment yields the greater return over 4 years: 8.25% compounded quarterly or 8.3% compounded semiannually?

$n=4$ $n=2$

Compounded quarterly:

$A = P(1+\frac{r}{n})^{nt}$

$P=6000$ $r=.0825$ $t=4$ $n=4$

$A = 6000(1+\frac{.0825}{4})^{4(4)}$

$= 6000(1.0206)^{16}$

$= \$8317.84$

Compounded Semiannually $n$

$P=6000$ $r=.083$ $t=4$ $n=2$

$A = 6000(1+\frac{.083}{2})^{2(4)}$

$= 6000(1.0415)^{8}$

$= \$8306.64$

8.25% compounded quartely is greater

*The function*

$$f(x) = \frac{90}{1 + 270e^{-0.122x}}$$

*models the percentage, $f(x)$, of people $x$ years old with some coronary heart disease. Use this function to solve Exercises 61–62. Round answers to the nearest tenth of a percent.*

55. Evaluate $f(30)$ and describe what this means in practical terms.

--------HOMEWORK------------

56. Evaluate $f(70)$ and describe what this means in practical terms.

$$f(70) = \frac{90}{1+270e^{-.122(70)}}$$

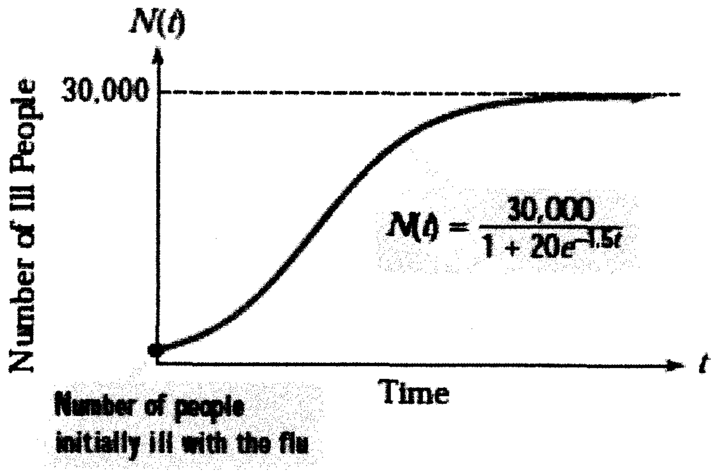
$$= 85.5\%$$

at the age of 70 approximately 85.5% of people will have some heart disease.

57. The function

$$N(t) = \frac{30{,}000}{1 + 20e^{-1.5t}}$$

describes the number of people, $N(t)$, who become ill with influenza $t$ weeks after its initial outbreak in a town with 30,000 inhabitants. The horizontal asymptote in the graph indicates that there is a limit to the epidemic's growth.




a. How many people became ill with the flu when the epidemic began? (When the epidemic began, $t = 0$.)

$t = 0$

$\frac{30000}{1+20(e^{-1.5(0)})}$

$\frac{30000}{21}$

b. How many people were ill by the end of the third week?

14828 people

c. Why can't the spread of an epidemic simply grow indefinitely? What does the horizontal asymptote shown in the graph indicate about the limiting size of the population that becomes ill?

b) $N(3) = \frac{30000}{1+20e^{-1.5(3)}} = 24{,}546$ People

c) there are only 30,000 inhabitants so eventually you run out of people.