

THE CHANGE-OF-BASE FORMULA

For any logarithm bases a and b and any positive number M ,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

EXAMPLE 8 Find $\log_4 7$ using common logarithms.

Let $a = 10$, $b = 4$, and $M = 7$. Then we substitute into the change-of-base formula:

$$\begin{aligned} \log_b M &= \frac{\log_a M}{\log_a b} \\ \log_4 7 &= \frac{\log_{10} 7}{\log_{10} 4} && \text{Substituting 10 for } a, \\ &= \frac{\log 7}{\log 4} && \text{4 for } b, \text{ and 7 for } M \\ &\approx 1.4037. \end{aligned}$$

To check, we use a calculator with a power key y^x or \wedge to verify that

$$4^{1.4037} \approx 7.$$

We can also use base e for a conversion.

EXAMPLE 9 Find $\log_4 7$ using natural logarithms.

$$\begin{aligned} \log_b M &= \frac{\log_a M}{\log_a b} \\ \log_4 7 &= \frac{\log_e 7}{\log_e 4} && \text{Substituting } e \text{ for } a, \\ &= \frac{\ln 7}{\ln 4} && \text{4 for } b, \text{ and 7 for } M \\ &\approx 1.4037 && \text{Note that this is the same answer as Example 8.} \end{aligned}$$

EXAMPLE 10 Find $\log_5 29$ using natural logarithms.

Substituting e for a , 5 for b , and 29 for M , we have

$$\begin{aligned} \log_5 29 &= \frac{\log_e 29}{\log_e 5} && \text{Using the change-of-base formula} \\ &= \frac{\ln 29}{\ln 5} \approx 2.0922. \end{aligned}$$

In Exercises 61-68, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

62. $\log_6(17)$

$$\log_6(17) = \frac{\log(17)}{\log(6)} \approx 1.5812$$

66. $\log_{0.3}(19)$

$$\log_{0.3}(19) = \frac{\log(19)}{\log(.3)} \approx -2.4456$$

Homework: 14.4 61-67 odd

In Exercises 61–68, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

61. $\log_5 13$

62. $\log_6 17$

63. $\log_{14} 87.5$

64. $\log_{16} 57.2$

65. $\log_{0.1} 17$

66. $\log_{0.3} 19$

67. $\log_{\pi} 63$

68. $\log_{\pi} 400$

Answers:

61. 1.5937 **63.** 1.6944 **65.** -1.2304 **67.** 3.6193