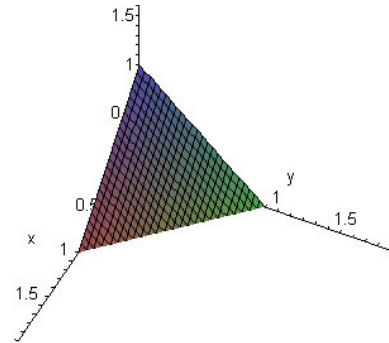


Surface Integrals 16.7 Part 1 non-parameterized surfaces

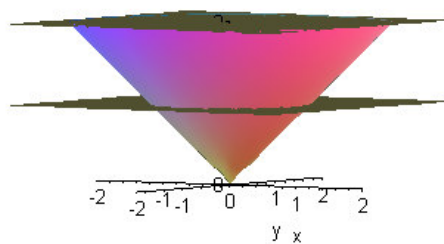
$$\iint_S f(x, y, z) dS = \iint_D f(x, y, g(x, y)) \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1} dA$$

Surface: $g(x, y) = z$

Example: Compute $\iint_S xz dS$ if the surface is the part of the plane given by:
 $x + y + z = 1$ in the first octant.



Example: Compute $\iint_S y^2 z^2 dS$ where S is the part of the cone $z = \sqrt{x^2 + y^2}$
 between the planes $z = 1$ and $z = 2$.

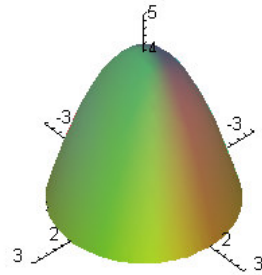


Surface Integral of a Vector Field – “Flux Integral”

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} dS = \iint_D \left(-P \frac{\partial g}{\partial x} - Q \frac{\partial g}{\partial y} + 1 \right) dA$$

where \vec{n} is an upward normal.

Example: Compute $\iint_S \vec{F} \cdot d\vec{S}$. Let $\vec{F}(x, y, z) = xy\vec{i} + yz\vec{j} + zx\vec{k}$. Bound the paraboloid $z = 4 - x^2 - y^2$ by the xy -plane.



Example: A fluid with density 1200 flows with a velocity vector of $\vec{v}(x, y, z) = y\vec{i} + \vec{j} + z\vec{k}$. Find the rate of flow upward through the paraboloid $z = 9 - \frac{1}{4}(x^2 + y^2)$; where $x^2 + y^2 \leq 36$

