Mat 241 Homework Set 9 – Due _____

Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

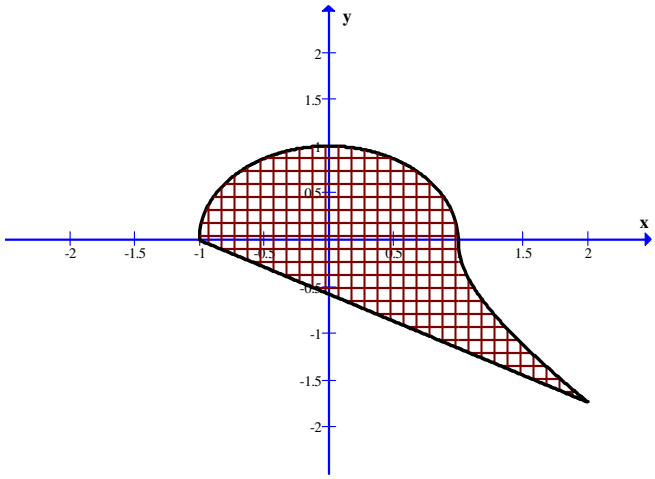
Mechanics:

#1. Neatly sketch the region, *R*, bounded by the y axis, $y = 2x$ and $x + 1 = (y - 1)^4$. Then set-up double integrals of types I and II that would compute the region's area.

#2. The region shown below is the union of a type I and type II region and was created out of the following three functions:

$$x^2 + y^2 = 1; x^2 - y^2 = 1; y = -\frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{3}.$$

Compute the double integral over the region with $f(x, y) = yx$.



#3. Sketch the region of integration for: $\int_{\theta=\frac{\pi}{6}}^{\frac{3\pi}{4}} \int_{r=1}^{2\sin\theta} f(r,\theta) r\,dr\,d\theta$

#4. Evaluate the iterated integrals by first converting them to polar coordinates.

A. $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx$ **B.** $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2\,dx\,dy$

#5. Consider the three double integrals shown below. Sketch the three regions of integration on the same graph. Using your sketch, create an equivalent polar representation and evaluate exactly.

$$\int_{\frac{1}{\sqrt{2}}}^{1}\int_{\sqrt{1-x^2}}^{x} xydydx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xydydx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xydydx$$

#6.Typically, texts use very nice set-ups for polar integration problems in the sense that intersection points are easily computed. Let's not go that route.

A. **Graph the polar equations $r = 1+\cos\theta$ and $r = 4\cos\theta$ carefully and accurately.**
B. **Set-up a double integral to find the area of the region that lies outside the cardioid and still inside the circle.**
C. **Evaluate this integral exactly. *Enjoy!***

<u>Concepts and Applications.</u>

#6. A sprinkler distributes water in a circular pattern. It supplies water to a depth of e^{-r} feet per hour at a distance r feet from the sprinkler.

A. **What is the total amount of water supplied per hour to a circular region centered at the sprinkler of radius R?**
B. **Determine an expression for the average amount of water per hour per square foot supplied to the region. (see page 986).**

#7. Find the volume of an ice-cream cone bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$. Supply graphics if you wish. Yummy.

#8. Use a double integral in polar coordinates to prove that the volume of a sphere of radius a, is $\frac{4\pi a^3}{3}$ (the formula Mrs. Snaggletooth told you in 7th grade).