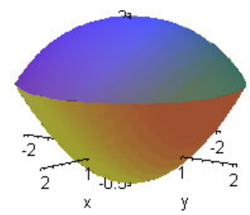
Mat 241 Homework Set 14 – Due ______________

Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are implemented, do so appropriately.**

Mechanics: - Surface Integrals & Stokes' Theorem

#1. We are given a surface whose top is the upper hemisphere of $x^2 + y^2 + z^2 = 4$ and whose bottom is the paraboloid $2z = -1 + x^2 + y^2$.



A. Find the surface area of the shown closed surface.

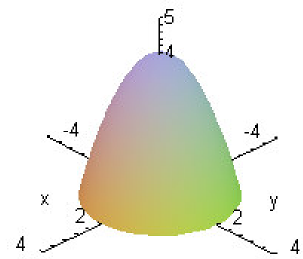
B. Given the field vector $\vec{F} = \dfrac{k}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}\langle x, y, z \rangle$, determine the total flux of $\vec{F}$ through the above closed surface.

Note: $Flux = \iint_{s} \vec{F} \cdot \vec{n} dS = \iint_{sphere} \vec{F} \cdot \vec{n} dS + \iint_{paraboloid} \vec{F} \cdot \vec{n} dS$

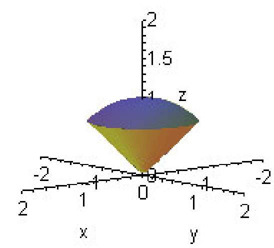
Practice with Stokes' Theorem

For problems 2 & 3 use Stoke's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

#2. $\vec{F}(x, y, z) = 3z\vec{i} + 4x\vec{j} + 2y\vec{k}$ where C is the boundary of the paraboloid shown. (ans.16π)



#3. $\vec{F}(x,y,z)=(z+\sin x)\vec{i}+(x+y^2)\vec{j}+(y+e^z)\vec{k}$ where C is the intersection of the sphere $\rho=1$ and the cone $z=\sqrt{x^2+y^2}$ with counterclockwise orientation looking down the positive z axis. (ans. $\frac{\pi}{2}$)



For problems 4, 5, 6 compute the line integral $\int_C \vec{F}\cdot d\vec{r}$ through elementary means and then verify your result by using Stokes' theorem.

#4. $\vec{F}(x,y,z)=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ and the surface is the portion of the cone $z=\sqrt{x^2+y^2}$ below the plane z = 1. (ans. 0)

#5. $\vec{F}(x,y,z)=x\vec{i}+y\vec{j}+z\vec{k}$ and the surface is the upper hemisphere $z=\sqrt{a^2-x^2-y^2}$. (ans. 0)

#6. $\vec{F}(x,y,z)=(z-y)\vec{i}+(z+x)\vec{j}-(x+y)\vec{k}$ and the surface is the portion of the paraboloid $z=9-x^2-y^2$ above the xy – plane. (ans. 18π)