Mat 241 Homework Set 8 – Due _____

Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

Mechanics:

For the following two integrals do the following:

A. **Sketch the region of integration for the two integrals shown.**
B. **Compute the integrals in 1 & 2 exactly.**
C. **Write each integral with the order of integration reversed and then compute each of the "new" integrals.**

1. $\int_0^2 \int_{\frac{y}{2}}^1 (x+y)\,dx\,dy$

2. $\int_0^1 \int_0^{\sqrt{1-y^2}} y\,dx\,dy$

For the following two integrals do the following:

D. **Sketch the region of integration.**
E. **Write each integral with the order of integration reversed.**
F. **Compute each of the "new" integrals. Hint: On #4 utilize a Taylor series using the first 6 terms to approximate one part of the region.**

3. $\int_0^{\ln 16} \int_{e^{\frac{x}{2}}}^4 \frac{1}{\ln y}\,dy\,dx$

4. $\int_0^1 \int_{y^2}^2 \frac{e^x}{\sqrt{x}}\,dx\,dy$

Concept development and applications.

#5. Use a CAS to compute the following two integrals showing that they are not the same. Why doesn't this contradict Fubini's Thoerem?

$$\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}dydx \quad \& \quad \int_0^1\int_0^1 \frac{x-y}{(x+y)^3}dxdy$$

Can you compute these by hand?(not required)

#6. The order of integration in a double integral is largely a matter of choice but sometimes the order can be the difference between a straightforward evaluation as opposed to a very difficult if not impossible evaluation. Consider the function $f(x,y) = x\cos(xy)$ **on** $R = \left[0,\frac{\pi}{2}\right]\times[0,1]$**.**

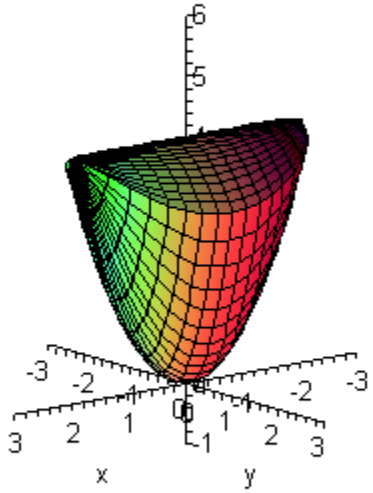
A. **Integrate the function over the specified region with respect to y first (i.e. dydx)**
B. **Integrate the function over the specified region with respect to x first and indicate when the difficulty arises. (i.e. dxdy).**

#7. A rectangular plate of sides lengths *a* and *b* is subjected to a normal force (that is perpendicular to the plate). The pressure, *p*, at any point on the plate is proportional to the square of the distance of that point from one corner. Find the total force on the plate [Note: Pressure is force per unit area].

#8. Find the volume of the solid bounded by the paraboloid $z = 9x^2 + y^2$ **above, by the plane z = 0 below, and laterally by the planes x = 0, y = 0, x = 3, and y = 2. Sketch the Region in the xy – plane and indicate your directions of integrations**

#9. Find the volume of the solid bounded by the two surfaces. Sketch the Region in the xy – plane and indicate your directions of integrations.

$$z = x^2 + 3y^2 \ \& \ z = 4 - y^2 .$$



#10. Find the volume of the solid formed by the two paraboloids $z = x^2 + 3y^2$ & $z = 9 - 2x^2 - y^2$. Sketch the Region in the xy – plane and indicate your directions of integrations.